Equilibria for Insurance Covers of Natural Catastrophes on Heterogeneous Regions

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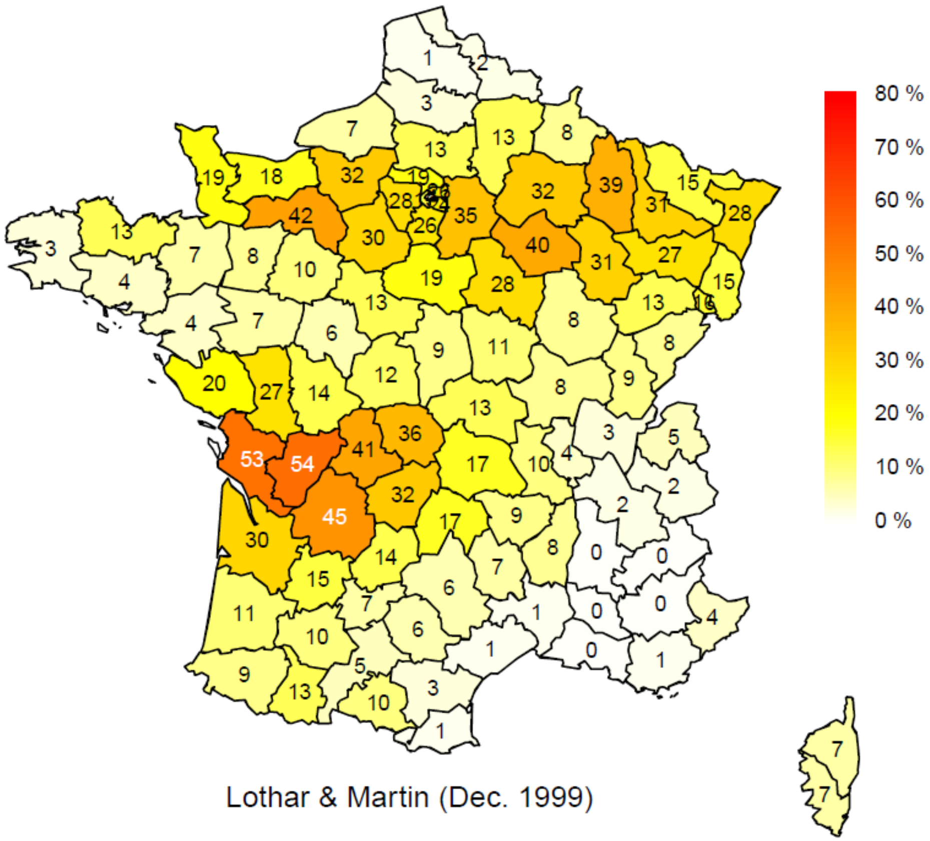
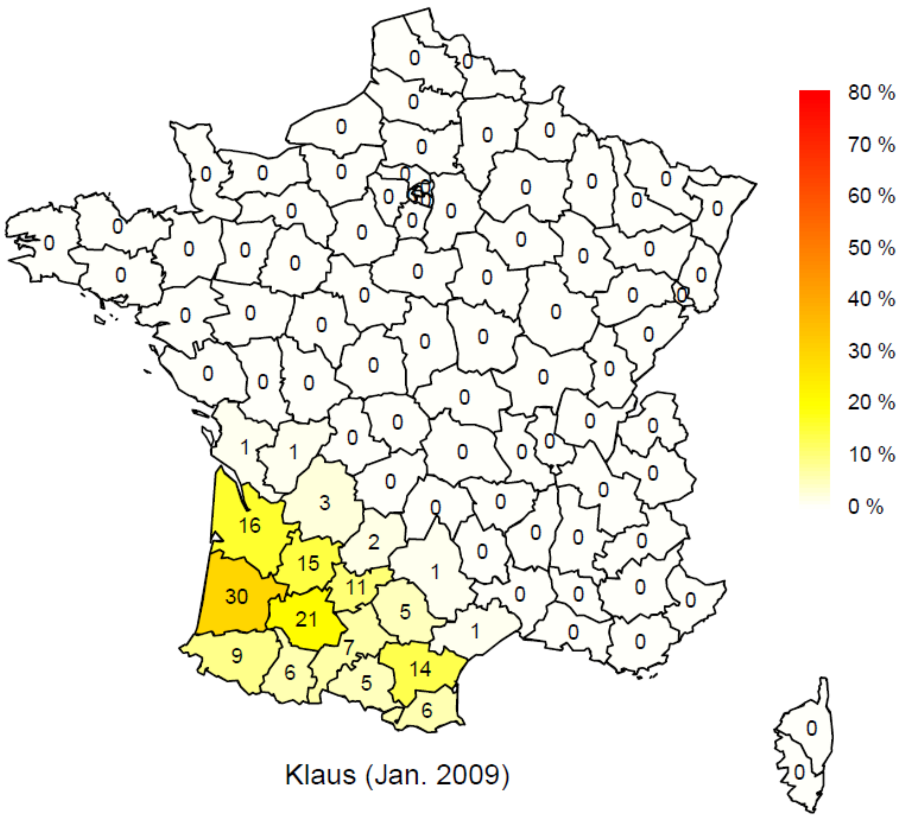
http://freakonometrics.hypotheses.org

Major (Winter) Storms in France



Lothar & Martin (Dec. 1999)



Klaus (Jan. 2009)

Proportion of insurance policy that did claim a loss after storms, for a large insurance company in France (~1.2 million household policies)

Demand for Insurance

An agent purchases insurance if

$$\underbrace{\mathbb{E}[u(\omega - X)]}_{\text{no insurance}} \leq \underbrace{u(\omega - \alpha)}_{\text{insurance}}$$

i.e.

$$\underbrace{p \cdot u(\omega - l) + [1 - p] \cdot u(\omega - 0)}_{\text{no insurance}} \leq \underbrace{u(\omega - \alpha)}_{\text{insurance}}$$

i.e.

$$\underbrace{\mathbb{E}[u(\omega - X)]}_{\text{no insurance}} \leq \underbrace{\mathbb{E}[u(\omega - \alpha - l + I)]}_{\text{insurance}}$$

Doherty & Schlessinger (1990) considered a model which integrates possible bankruptcy of the insurance company, but as an exogenous variable. Here, we want to make ruin endogenous.

Notations

$$Y_i = \begin{cases} 0 \text{ if agent } i \text{ claims a loss} \\ 1 \text{ if not} \end{cases}$$

Let $N = Y_1 + \cdots + Y_n$ denote the number of insured claiming a loss, and $X = N/n$ denote the proportions of insured claiming a loss, $F(x) = \mathbb{P}(X \leq x)$.

$$\mathbb{P}(Y_i = 1) = p \text{ for all } i = 1, 2, \cdots, n$$

Assume that agents have identical wealth ω and identical utility functions $u(\cdot)$.

Further, insurance company has capital $C = n \cdot c$, and ask for premium α.

Private insurance companies with limited liability

Consider $n = 5$ insurance policies, possible loss \$1,000 with probability 10%.
Company has capital $C = 1,000$.

	Ins. 1	Ins. 1	Ins. 3	Ins. 4	Ins. 5	Total
Premium	100	100	100	100	100	500
Loss	-	1,000	-	1,000	-	2,000
Case 1: *insurance company with limited liability*						
indemnity	-	750	-	750	-	1,500
loss	-	-250	-	-250	-	-500
net	-100	-350	-100	-350	-100	-1000

Possible government intervention

	Ins. 1	Ins. 1	Ins. 3	Ins. 4	Ins. 5	Total
Premium	100	100	100	100	100	500
Loss	-	1,000	-	1,000	-	2,000
Case 2: *possible government intervention*						
Tax	-100	100	100	100	100	500
indemnity	-	1,000	-	1,000	-	2,000
net	-200	-200	-200	-200	-200	-1000

(note that it is a zero-sum game).

A one region model with homogeneous agents

Let $U(x) = u(\omega + x)$ and $U(0) = 0$.

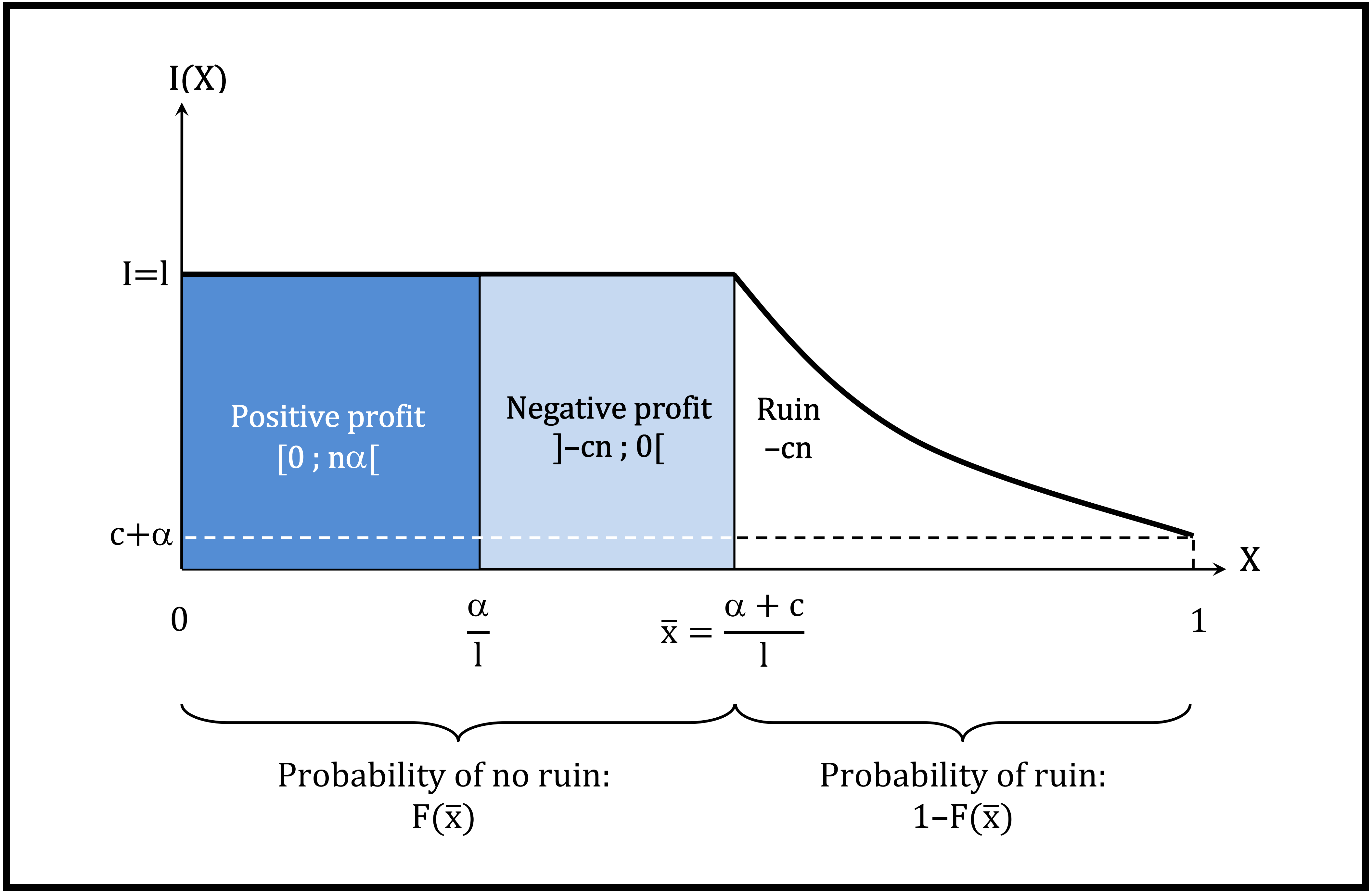
Private insurance companies with limited liability:

- the company has a positive profit if $N \cdot \ell \leq n \cdot \alpha$
- the company has a negative profit if $n \cdot \alpha \leq N \cdot \ell \leq C + n \cdot \alpha$
- the company is bankrupted if $C + n \cdot \alpha \leq N \cdot \ell$

$\Longrightarrow$ ruin of the insurance company if $X \geq \overline{x} = \dfrac{c + \alpha}{\ell}$

The indemnity function is

$$I(x) = \begin{cases} \ell \text{ if } x \leq \overline{x} \\ \dfrac{c + \alpha}{n} \text{ if } x > \overline{x} \end{cases}$$


I(X)
I=l
c+α
X
Positive profit
[0 ; nα[
Negative profit
]−cn ; 0[
Ruin
−cn
0
α/l
x̄ = (α + c)/l
1
Probability of no ruin:
F(x̄)
Probability of ruin:
1−F(x̄)

The objective function of the insured is V defined as

$$\mathbb{E}[\mathbb{E}(U(-\alpha - \text{loss})|X)]) = \int \mathbb{E}(U(-\alpha - \text{loss})|X = x)dF(x)$$

where $\mathbb{E}(U(-\alpha - \text{loss})|X = x)$ is equal to

$$\mathbb{P}(\text{claim a loss}|X = x) \cdot U(\alpha - \text{loss}(x)) + \mathbb{P}(\text{no loss}|X = x) \cdot U(-\alpha)$$

i.e.

$$\mathbb{E}(U(-\alpha - \text{loss})|X = x) = x \cdot U(-\alpha - \ell + I(x)) + (1 - x) \cdot U(-\alpha)$$

so that

$$V = \int_0^1 [x \cdot U(-\alpha - l + I(x)) + (1 - x) \cdot U(-\alpha)]dF(x)$$

that can be written

$$V = U(-\alpha) - \int_0^1 x[U(-\alpha) - U(-\alpha - \ell + I(x))]f(x)dx$$

An agent will purchase insurance if and only if $V > p \cdot U(-l)$.

with government intervention (or mutual fund insurance), the tax function is

$$T(x) = \begin{cases} 0 \text{ if } x \leq \overline{x} \\ \dfrac{N\ell - (\alpha + c)n}{n} = X\ell - \alpha - c \text{ if } x > \overline{x} \end{cases}$$

Then

$$V = \int_0^1 [x \cdot U(-\alpha - T(x)) + (1 - x) \cdot U(-\alpha - T(x))] dF(x)$$

i.e.

$$V = \int_0^1 U(-\alpha + T(x)) dF(x) = F(\overline{x}) \cdot U(-\alpha) + \int_{\overline{x}}^1 U(-\alpha - T(x)) dF(x)$$

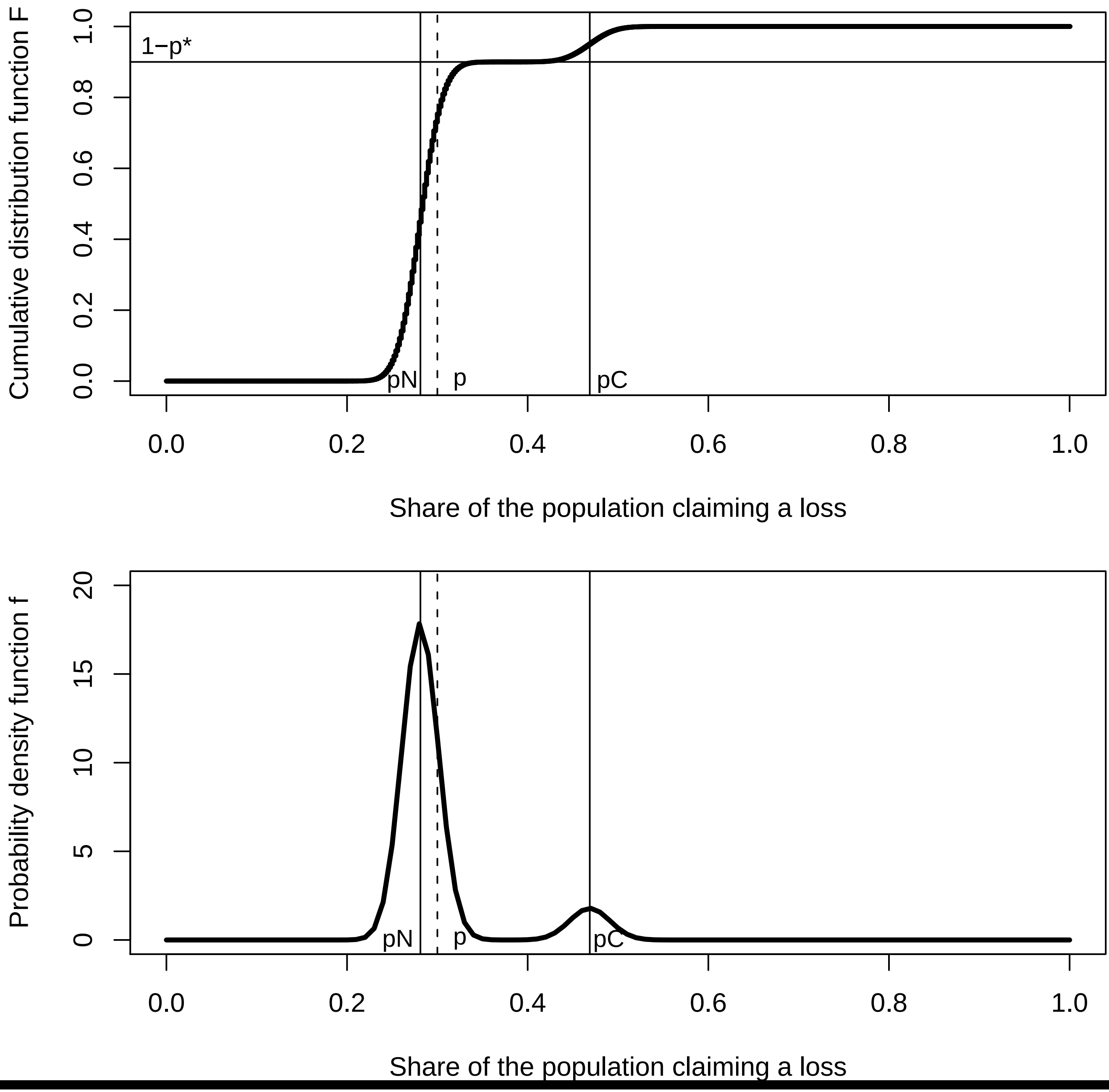
A common shock model for natural catastrophes risks

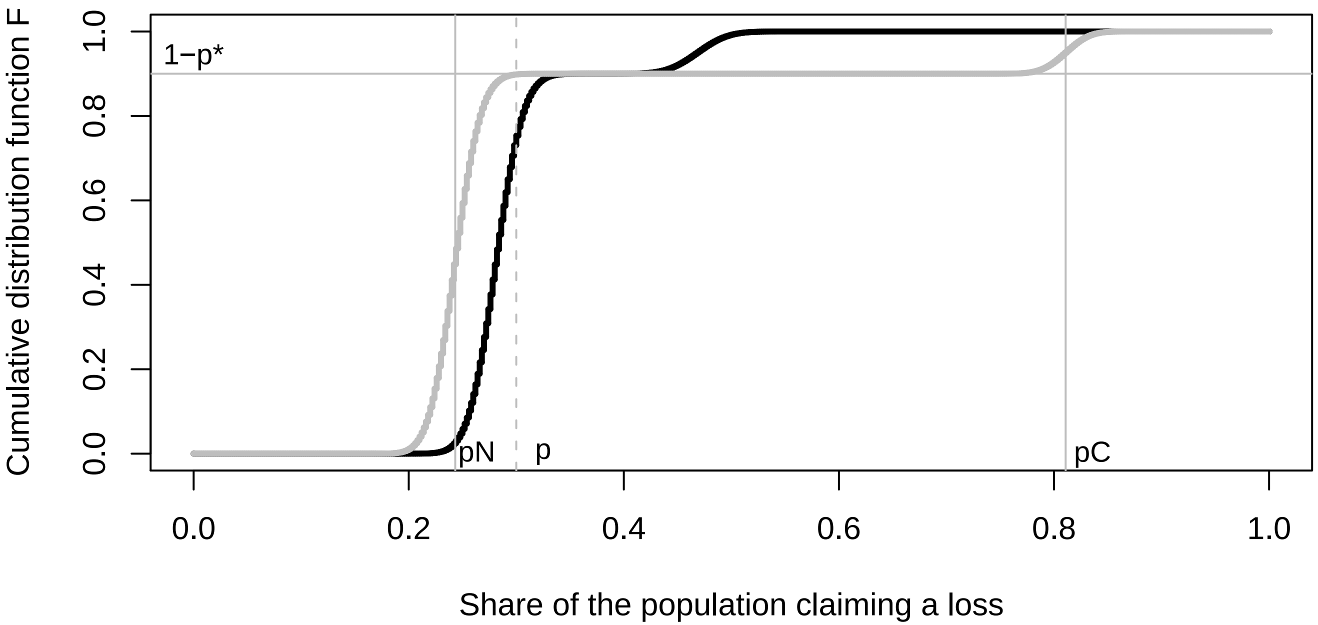
Consider a possible natural castrophe, modeled as an heterogeneous latent variable Θ, such that given Θ, the Y_i's are independent, and

$$\begin{cases} \mathbb{P}(Y_i = 1|\Theta = \text{ Catastrophe}) = p_C \\ \mathbb{P}(Y_i = 1|\Theta = \text{ No Catastrophe}) = p_N \end{cases}$$

Let $p^\star = \mathbb{P}(\text{Cat})$. Then the distribution of X is

$$\begin{aligned} F(x) &= \mathbb{P}(N \leq [nx]) = \mathbb{P}(N \leq k|\text{No Cat}) \times \mathbb{P}(\text{No Cat}) + \mathbb{P}(N \leq k|\text{Cat}) \times \mathbb{P}(\text{Cat}) \\ &= \sum_{j=0}^{k} \binom{n}{j} \left[(p_N)^j (1-p_N)^{n-j}(1-p^*) + (p_C)^j (1-p_C)^{n-j} p^*\right] \end{aligned}$$


Cumulative distribution function F
1−p*
pN
p
pC
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function f
0
5
10
15
20
Share of the population claiming a loss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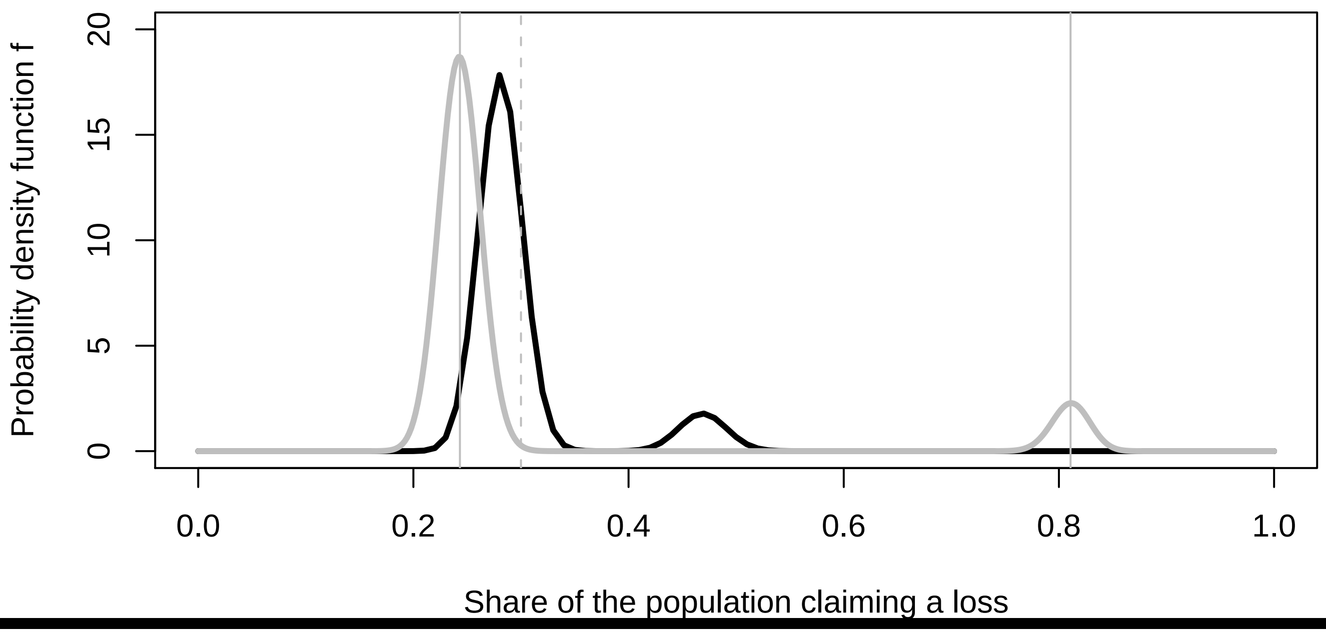
Probability density function f
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15
20
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss

Equilibriums in the Expected Utility framework

The expected profit of the insurance company is

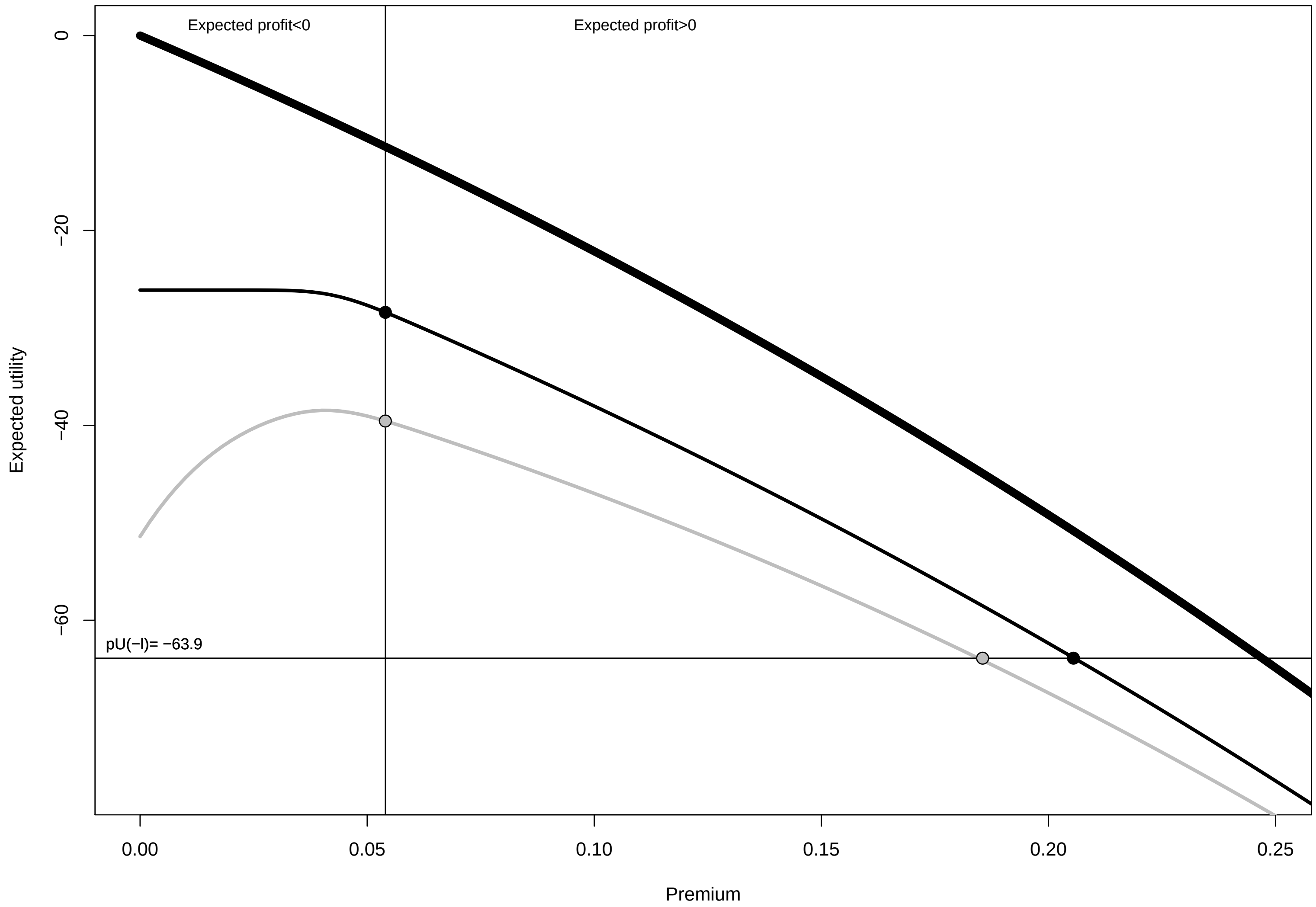
$$\Pi(\alpha, c) = \int_0^{\bar{x}} [n\alpha - xn\ell]\, dF(x) - [1 - F(\bar{x})]cn \tag{1}$$

A premium smaller than the pure premium can lead to a positive expected profit.

In Rothschild & Stiglitz (QJE, 1976) a positive profit was obtained if and only if $\alpha > p \cdot l$. Here companies have limited liabilities.

If agents are risk adverse, for a given premium α, their expected utility is always higher with government intervention.

Proof. Risk adverse agents look for mean preserving spread lotteries. □


Expected profit<0
Expected profit>0
pU(−I)= −63.9
0
−20
−40
−60
Expected utility
0.00
0.05
0.10
0.15
0.20
0.25
Premium

The two region model

Consider here a two-region chock model such that

- $\Theta = (0,0)$, no catastrophe in the two regions,
- $\Theta = (1,0)$, catastrophe in region 1 but not in region 2,
- $\Theta = (0,1)$, catastrophe in region 2 but not in region 1,
- $\Theta = (1,1)$, catastrophe in the two regions.

Let N_1 and N_2 denote the number of claims in the two regions, respectively, and set $N_0 = N_1 + N_2$.

The two region model

$$
\begin{aligned}
X_1 &\sim F_1(x_1|p_1,\delta_1) = F_1(x_1), \\
X_2 &\sim F_2(x_2|p_2,\delta_2) = F_2(x_2), \\
X_0 &\sim F_0(x_0|F_1,F_2,\theta) = F_0(x_0|p_1,p_2,\delta_1,\delta_2,\theta) = F_0(x_0),
\end{aligned}
$$

Note that there are two kinds of correlation in this model,

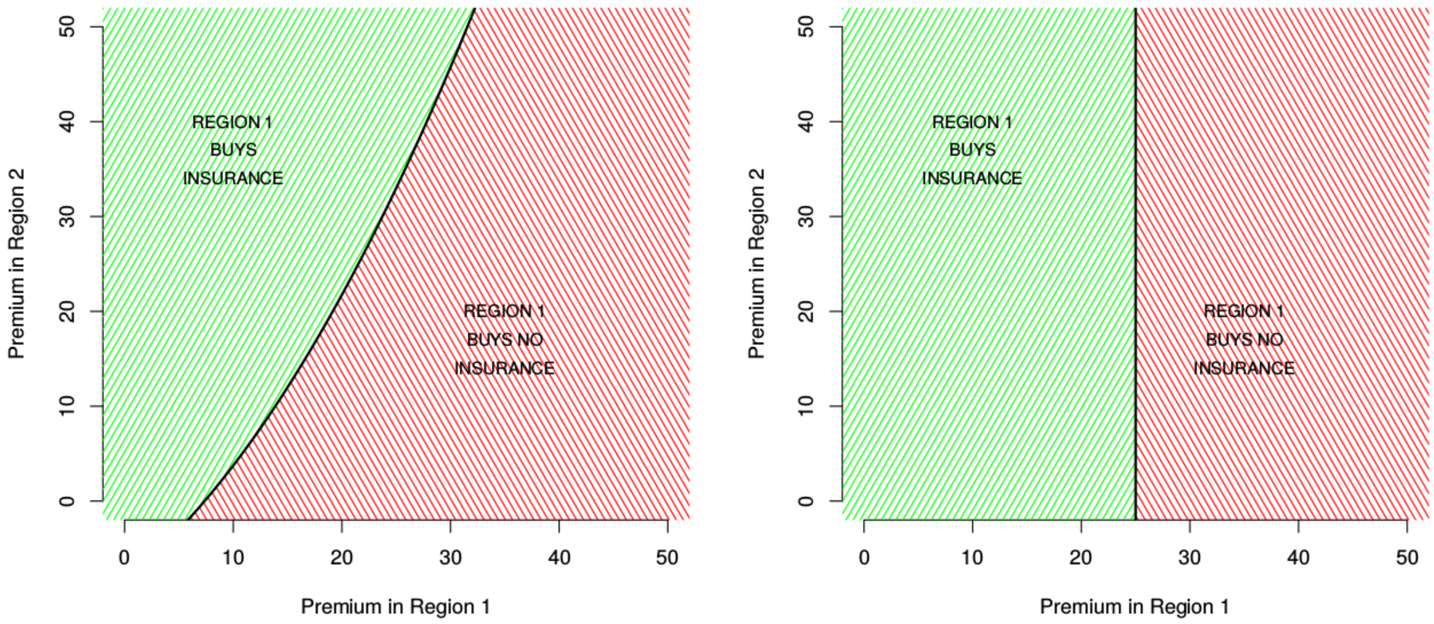
- a within region correlation, with coefficients δ_1 and δ_2
- a between region correlation, with coefficient δ_0

Here, $\delta_i = 1 - p_N^i/p_C^i$, where $i = 1, 2$ (Regions), while $\delta_0 \in [0,1]$ is such that

$$\mathbb{P}(\Theta = (1,1)) = \delta_0 \times \min\{\mathbb{P}(\Theta = (1,\cdot)), \mathbb{P}(\Theta = (\cdot,1))\} = \delta_0 \times \min\{p_1^\star, p_2^\star\}.$$

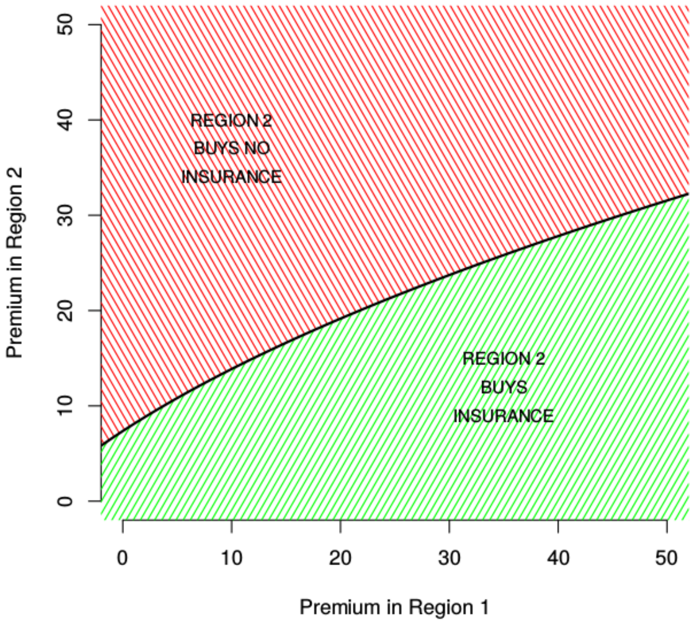
The two region model

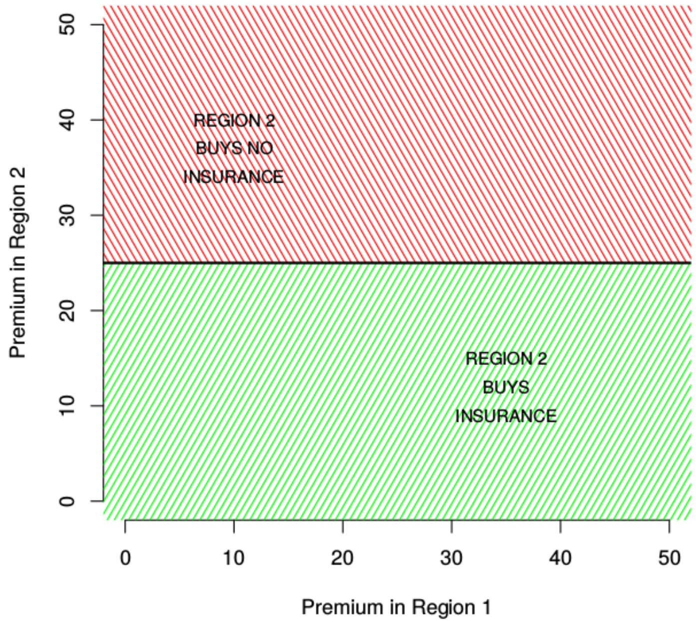
The following graphs show the decision in Region 1, given that Region 2 buy insurance (on the left) or not (on the right).



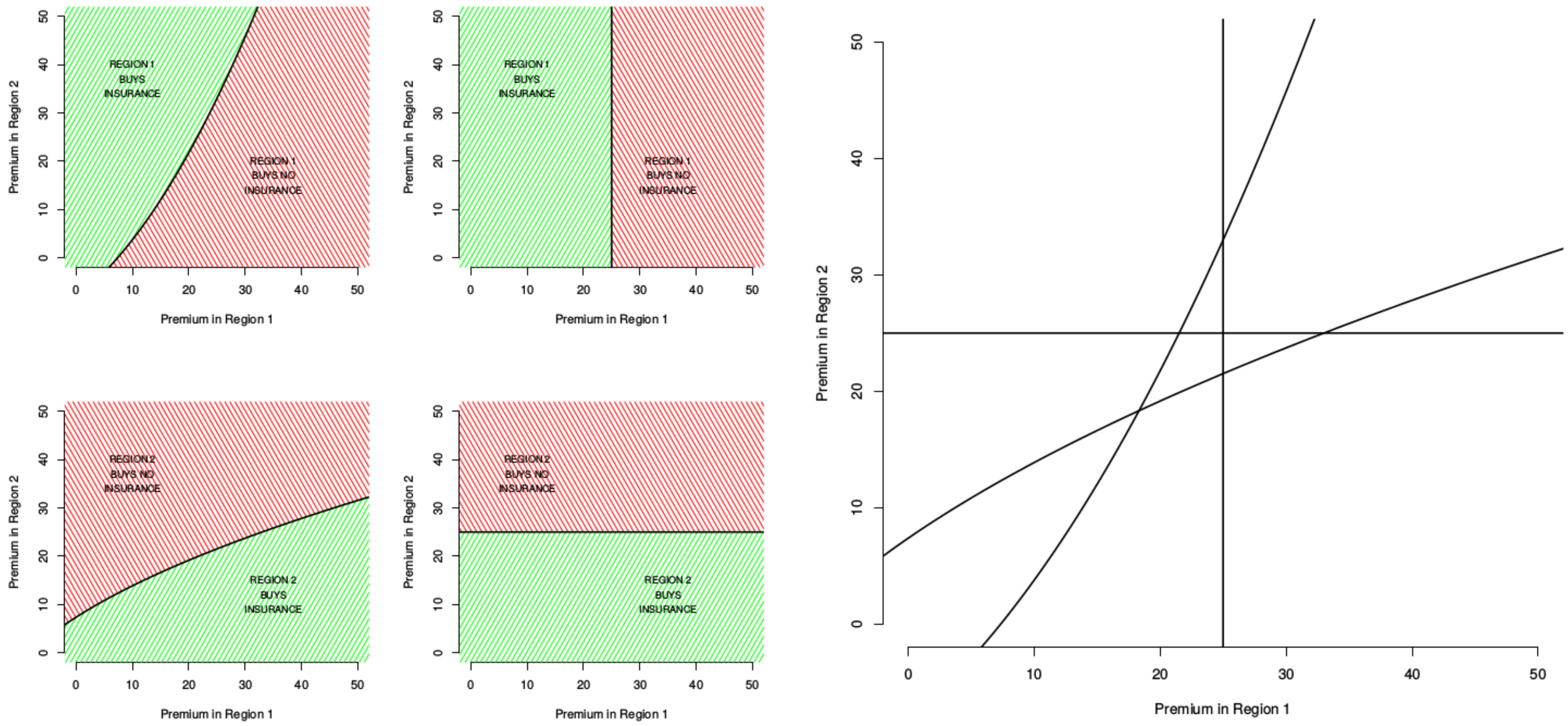
The two region model

The following graphs show the decision in Region 2, given that Region 1 buy insurance (on the left) or not (on the right).





In a Strong Nash equilibrium which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally.



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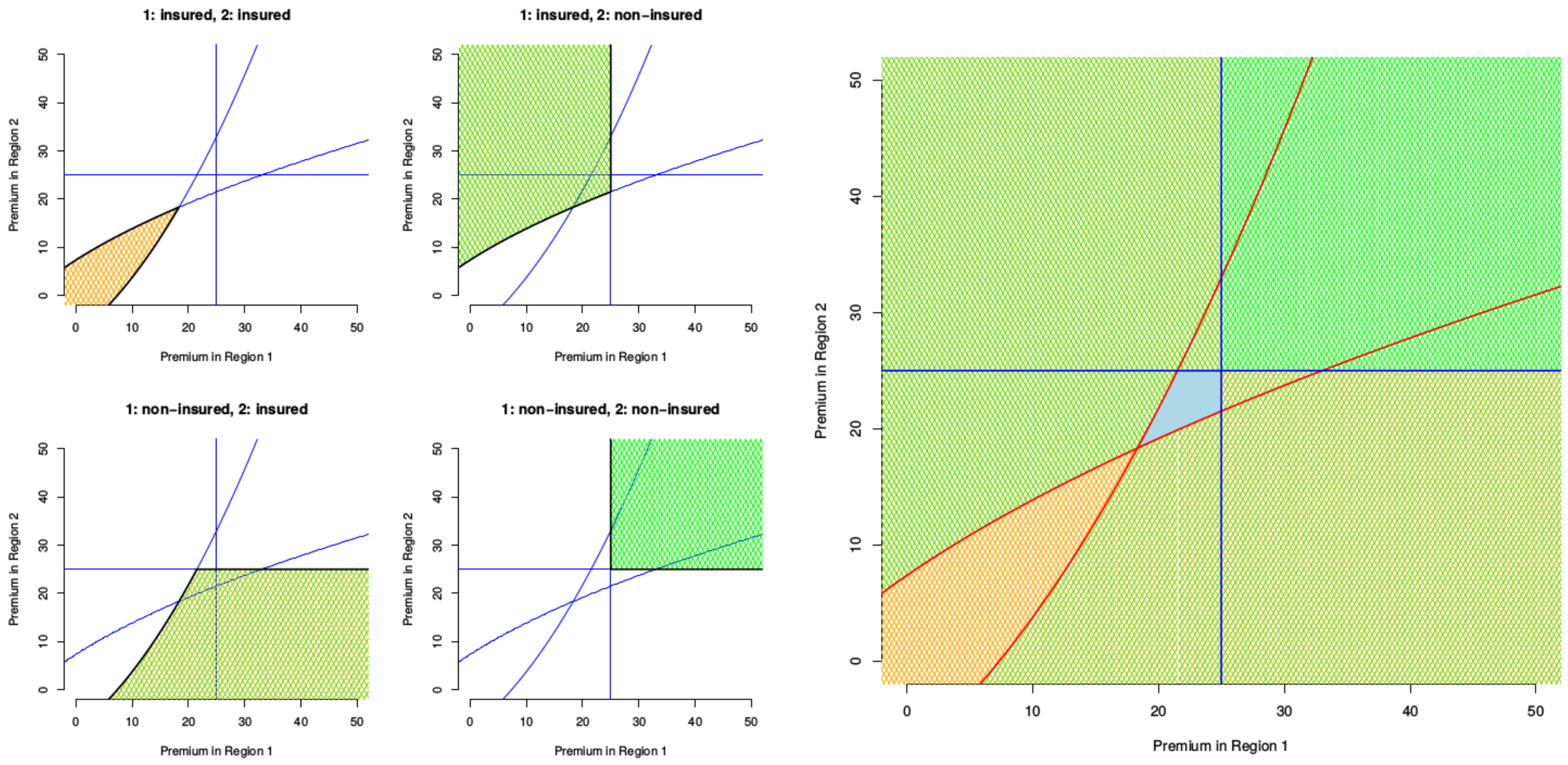
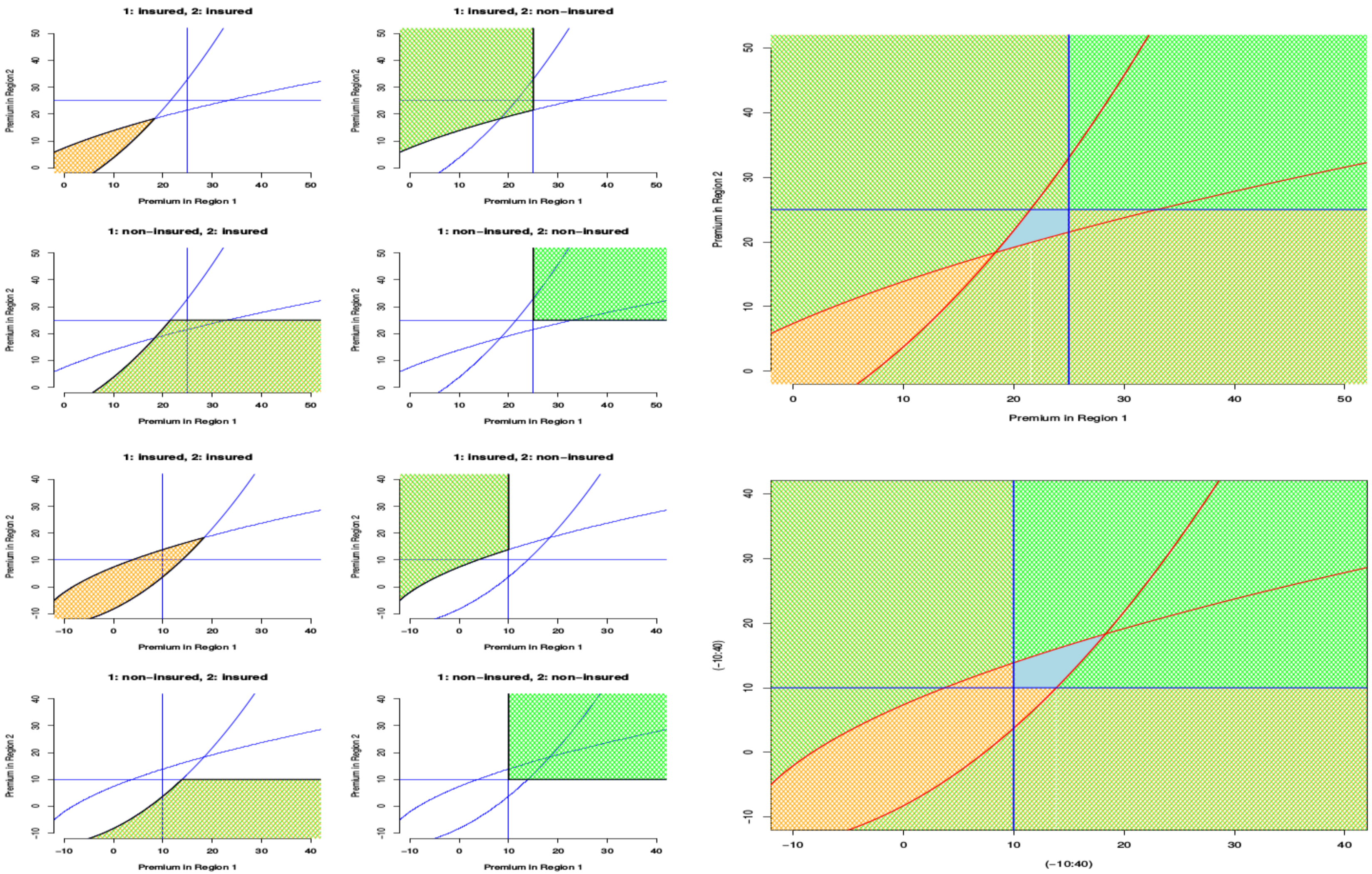


Possible Nash equilibriums



Possible Nash equilibriums