Insurance of Natural Catastrophes When Should Government Intervene ?

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http ://freakonometrics.blog.free.fr/

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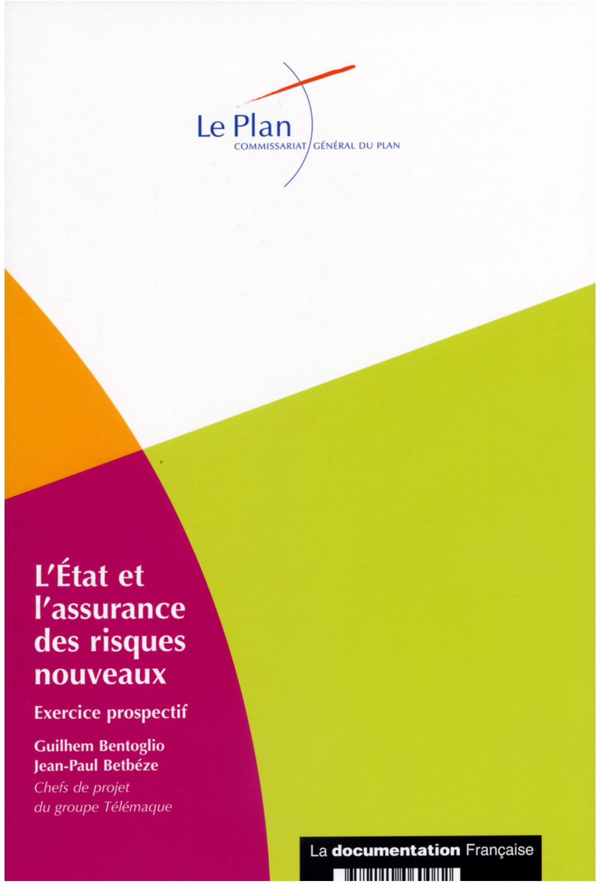
1 Introduction and motivation



Insurance is "*the contribution of the many to the misfortune of the few*".

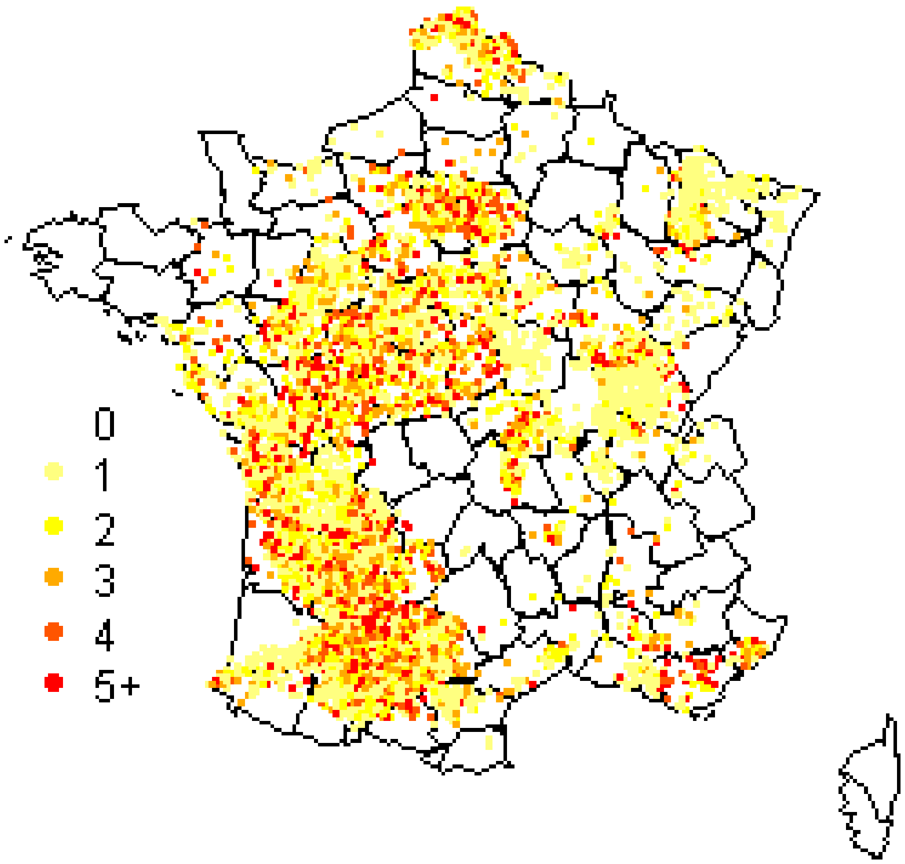
The TELEMAQUE working group, 2005.

Insurability requieres independence

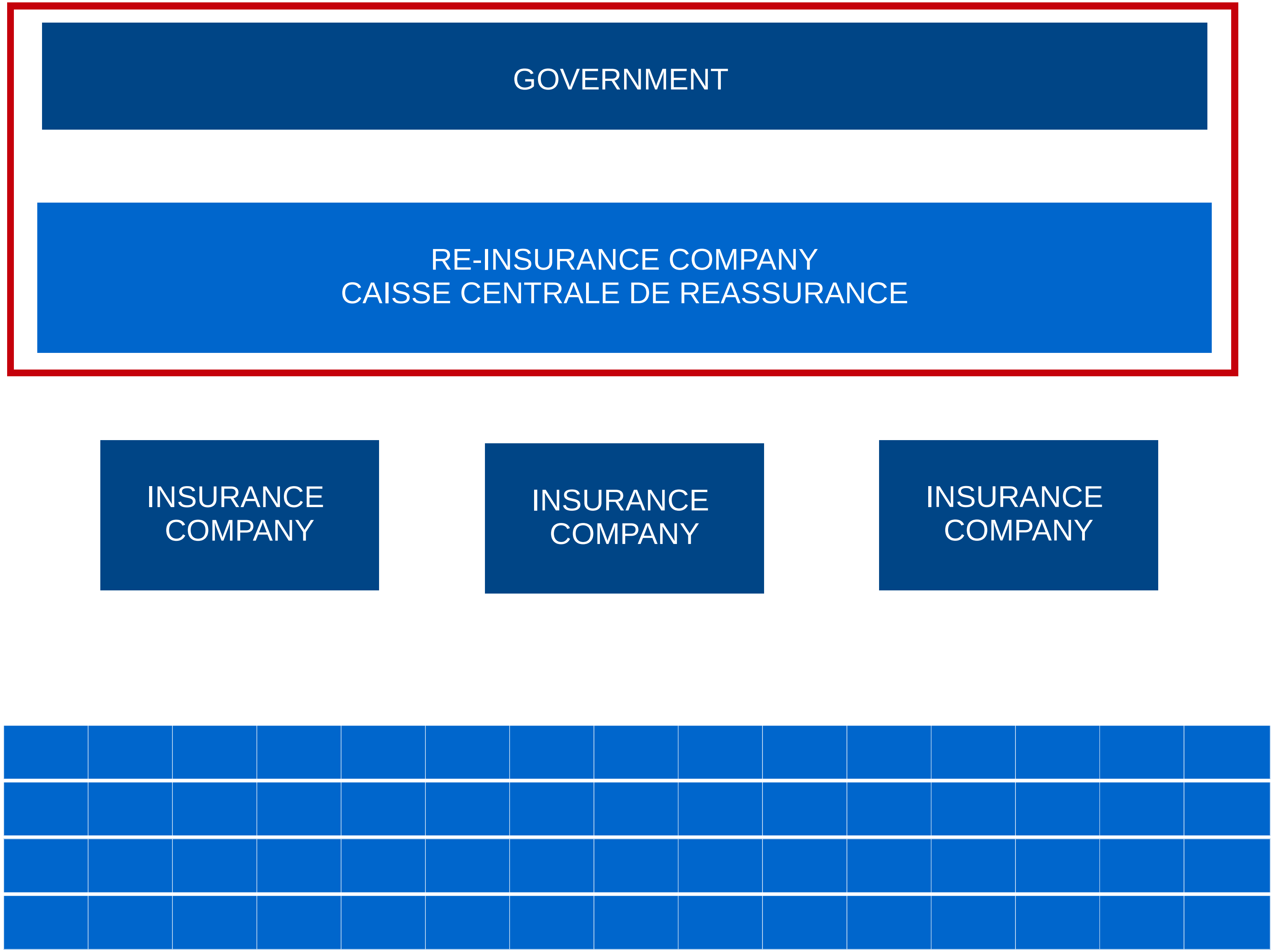
Cummins & Mahul (JRI, 2004) or C. (GP, 2008)

1.1 The French *cat nat* mecanism

$\Longrightarrow$ natural catastrophes means no independence



Drought risk frequency, over 30 years, in France.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RE-INSURANCE COMPANY
CAISSE CENTRALE DE REASSURANCE
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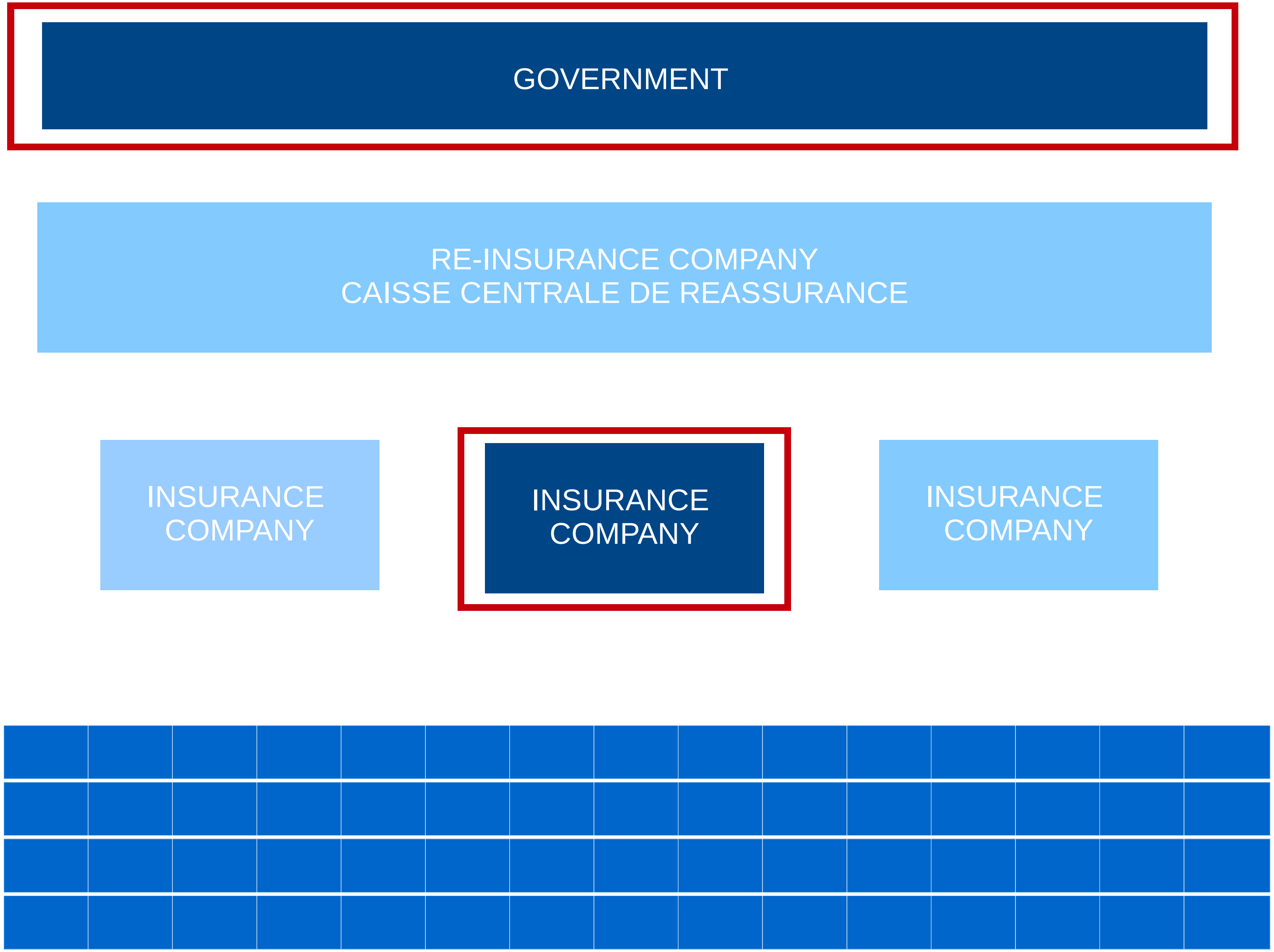
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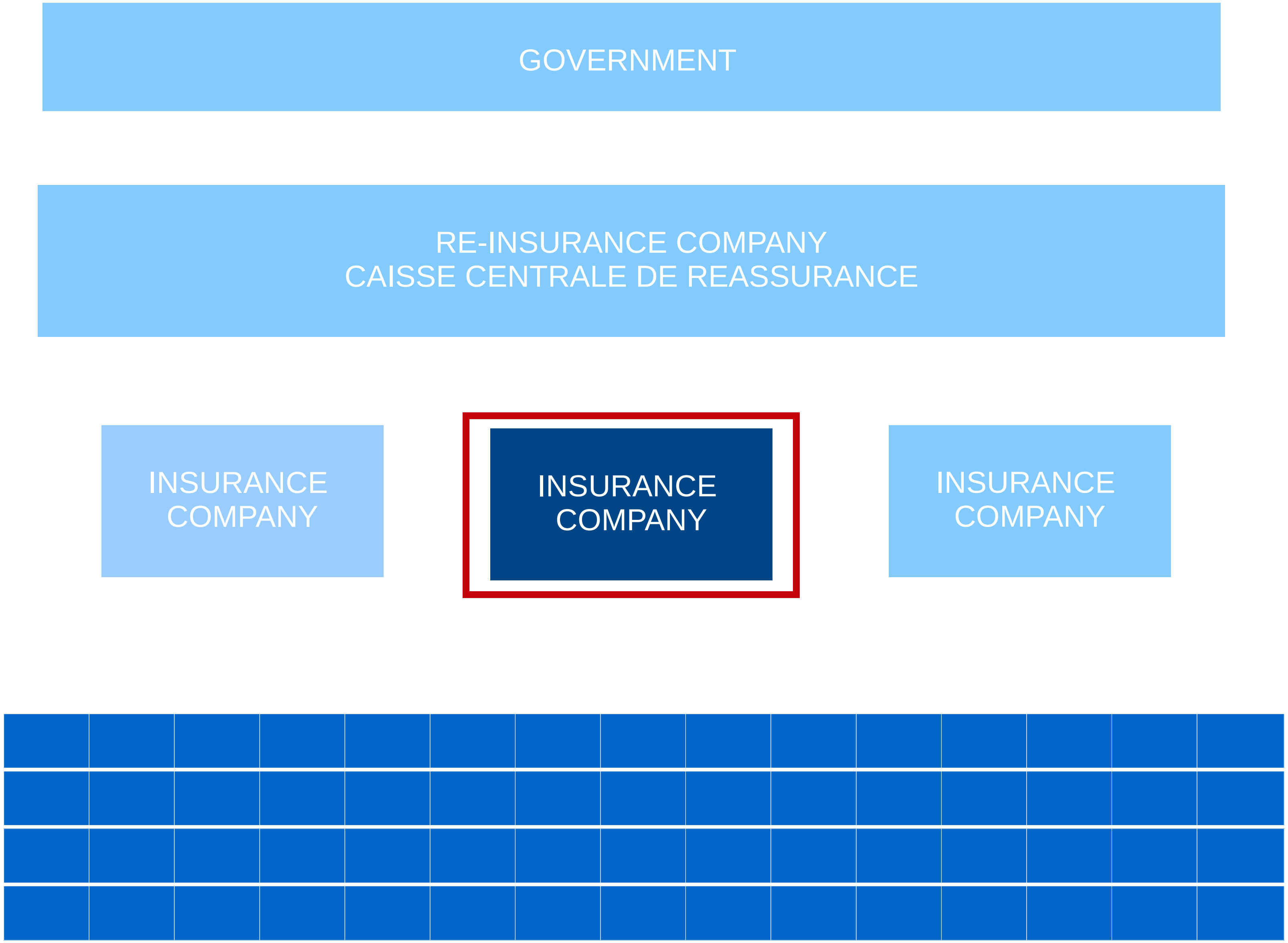
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INSURANCE COMPANY
INSURANCE COMPANY
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2 Demand for insurance

An agent purchases insurance if

$$\underbrace{\mathbb{E}[u(\omega - X)]}_{\text{no insurance}} \leq \underbrace{u(\omega - \alpha)}_{\text{insurance}}$$

i.e.

$$\underbrace{p \cdot u(\omega - l) + [1 - p] \cdot u(\omega - 0)}_{\text{no insurance}} \leq \underbrace{u(\omega - \alpha)}_{\text{insurance}}$$

i.e.

$$\underbrace{\mathbb{E}[u(\omega - X)]}_{\text{no insurance}} \leq \underbrace{\mathbb{E}[u(\omega - \alpha - l + I)]}_{\text{insurance}}$$

Doherty & Schlessinger (1990) considered a model which integrates possible bankruptcy of the insurance company, but as an exogenous variable. Here, we want to make ruin endogenous.

$$Y_i = \begin{cases} 0 \text{ if agent } i \text{ claims a loss} \\ 1 \text{ if not} \end{cases}$$

Let $N = Y_1 + \cdots + X_n$ denote the number of insured claiming a loss, and $X = N/n$ denote the proportions of insured claiming a loss, $F(x) = \mathbb{P}(X \leq x)$.

$$\mathbb{P}(Y_i = 1) = p \text{ for all } i = 1, 2, \cdots, n$$

Assume that agents have identical wealth ω and identical vNM utility functions $u(\cdot)$.

$\Longrightarrow$ exchangeable risks

Further, insurance company has capital $C = n \cdot c$, and ask for premium α.

2.1 Private insurance companies with limited liability

Consider $n = 5$ insurance policies, possible loss $\$1,000$ with probability 10%. Company has capital $C = 1,000$.

	Ins. 1	Ins. 1	Ins. 3	Ins. 4	Ins. 5	Total
Premium	100	100	100	100	100	500
Loss	-	1,000	-	1,000	-	2,000
Case 1 : *insurance company with limited liability*						
indemnity	-	750	-	750	-	1,500
loss	-	-250	-	-250	-	-500
net	-100	-350	-100	-350	-100	-1000

2.2 Possible government intervention

	Ins. 1	Ins. 1	Ins. 3	Ins. 4	Ins. 5	Total
Premium	100	100	100	100	100	500
Loss	-	1,000	-	1,000	-	2,000
Case 2 : *possible government intervention*						
Tax	-100	100	100	100	100	500
indemnity	-	1,000	-	1,000	-	2,000
net	-200	-200	-200	-200	-200	-1000

(note that it is a zero-sum game).

3 A one region model with homogeneous agents

Let $U(x) = u(\omega + x)$ and $U(0) = 0$.

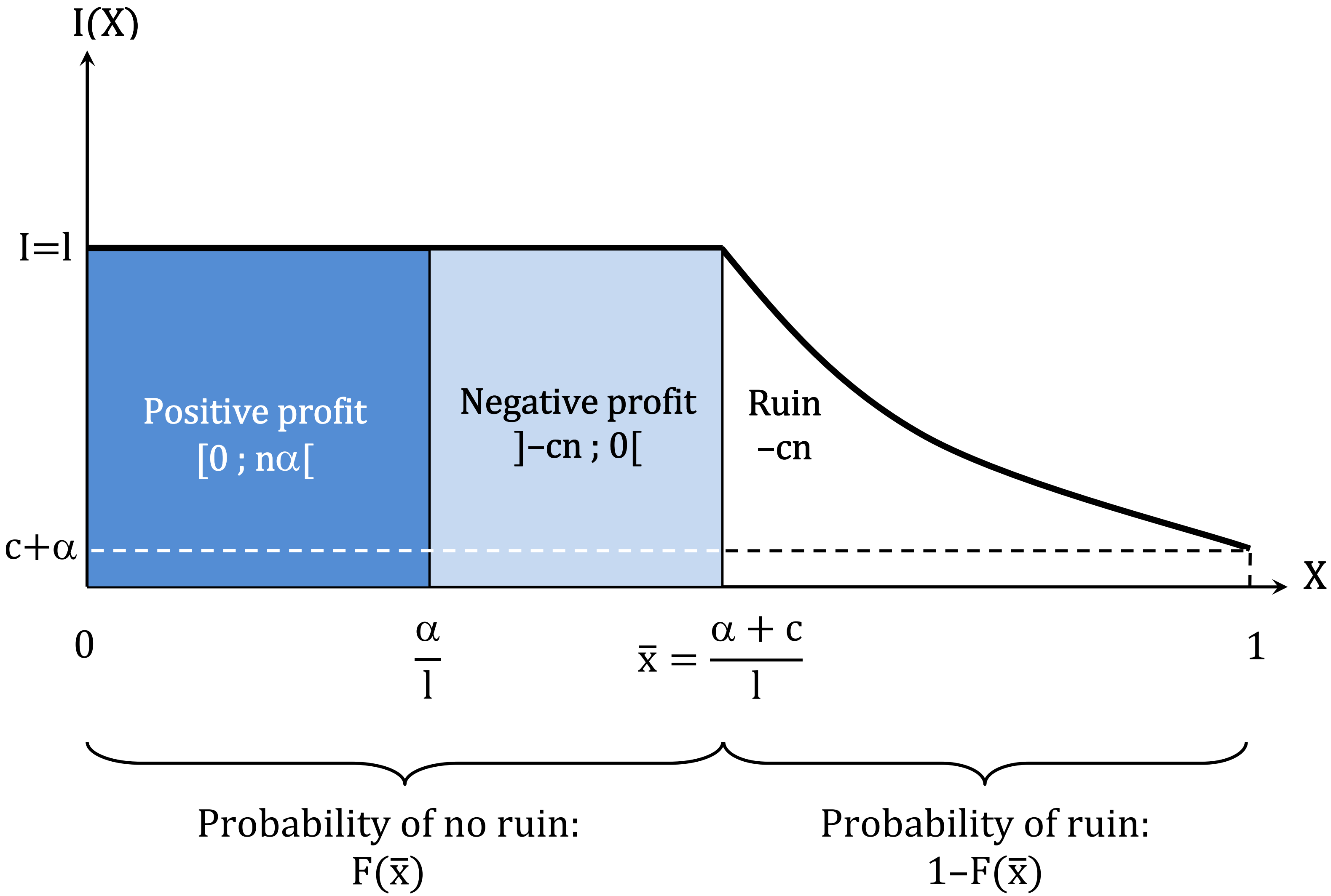
3.1 Private insurance companies with limited liability

- the company has a positive profit if $N \cdot l \leq n \cdot \alpha$
- the company has a negative profit if $n \cdot \alpha \leq N \cdot l \leq C + n \cdot \alpha$
- the company is bankrupted if $C + n \cdot \alpha \leq N \cdot l$

$\Longrightarrow$ ruin of the insurance company if $X \geq \overline{x} = \frac{c+\alpha}{l}$

The indemnity function is

$$I(x) = \begin{cases} l \text{ if } X \leq \overline{x} \\ \dfrac{c+\alpha}{n} \text{ if } X > \overline{x} \end{cases}$$


I(X)
I=l
c+α
Positive profit
[0 ; nα[
Negative profit
]−cn ; 0[
Ruin
−cn
X
0
α/l
x̄ = (α + c)/l
1
Probability of no ruin:
F(x̄)
Probability of ruin:
1−F(x̄)

Without ruin, the objective function of the insured is $V(\alpha, p, \delta, c)$ defined as $U(-\alpha)$. With possible ruin, it is

$$\mathbb{E}[\mathbb{E}(U(-\alpha - \text{loss})|X)]) = \int \mathbb{E}(U(-\alpha - \text{loss})|X = x) f(x)dx$$

where $\mathbb{E}(U(-\alpha - \text{loss})|X = x)$ is equal to

$$\mathbb{P}(\text{claim a loss}|X = x) \cdot U(\alpha - \text{loss}(x)) + \mathbb{P}(\text{no loss}|X = x) \cdot U(-\alpha)$$

i.e.

$$\mathbb{E}(U(-\alpha - \text{loss})|X = x) = x \cdot U(-\alpha - l + I(x)) + (1 - x) \cdot U(-\alpha)$$

so that

$$V = \int_0^1 [x \cdot U(-\alpha - l + I(x)) + (1 - x) \cdot U(-\alpha)] f(x)dx$$

that can be written

$$V = U(-\alpha) - \int_0^1 x[U(-\alpha) - U(-\alpha - l + I(x))] f(x)dx$$

And an agent will purchase insurance if and only if $V > p \cdot U(-l)$.

3.2 Distorted risk perception by the insured

We've seen that

$$V = U(-\alpha) - \int_0^1 x[U(-\alpha) - U(-\alpha - l + I(x))]f(x)dx$$

since $\mathbb{P}(Y_i = 1|X = x) = x$ (while $\mathbb{P}(Y_i = 1) = p$).

But in the model in the Working Paper (first version), we wrote

$$V = U(-\alpha) - \int_0^1 p[U(-\alpha) - U(-\alpha - l + I(x))]f(x)dx$$

i.e. the agent see x through the payoff function, not the occurence probability (which remains exogeneous).

3.3 Government intervention (or mutual fund insurance)

The tax function is

$$T(x) = \begin{cases} 0 \text{ if } X \leq \overline{x} \\ \dfrac{Nl - (\alpha + c)n}{n} = Xl - \alpha - c \text{ if } X > \overline{x} \end{cases}$$

Then

$$V = \int_0^1 [x \cdot U(-\alpha - T(x)) + (1 - x) \cdot U(-\alpha - T(x))] f(x) dx$$

i.e.

$$V = \int_0^1 U(-\alpha + T(x)) f(x) dx = F(\overline{x}) \cdot U(-\alpha) + \int_{\overline{x}}^1 U(-\alpha - T(x)) f(x) dx$$

4 The common shock model

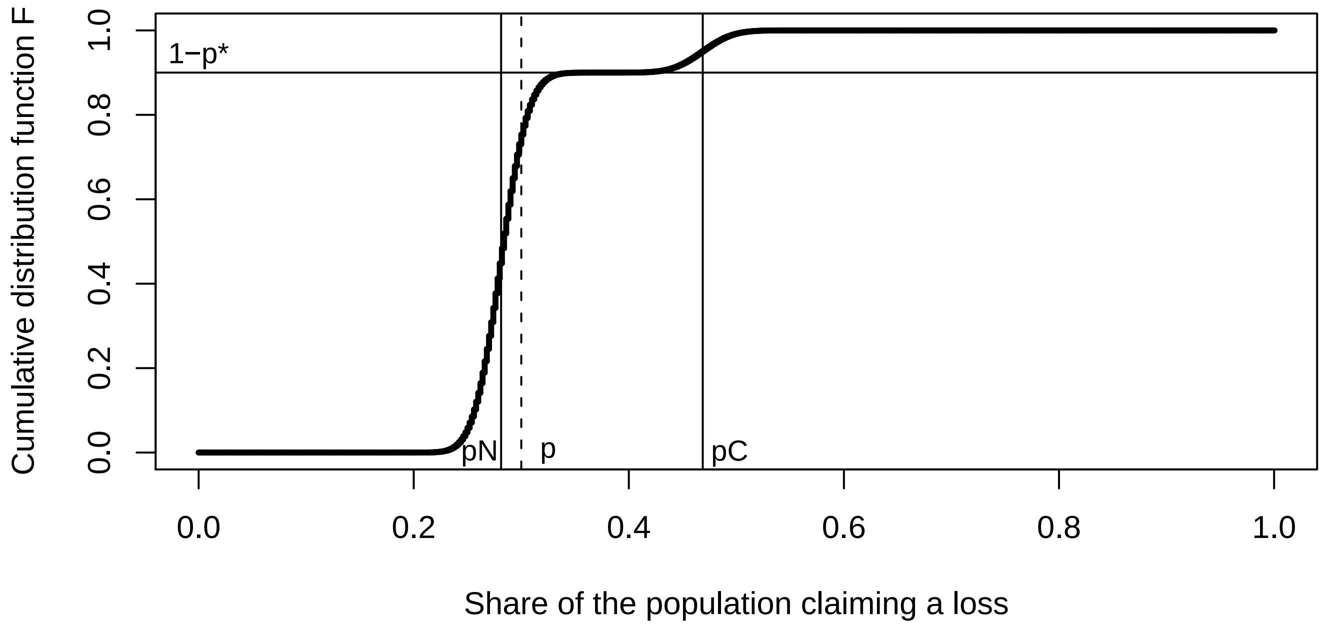
Consider a possible natural castrophe, modeled as an heterogeneous latent variable Θ, such that given Θ, the Y_i's are independent, and

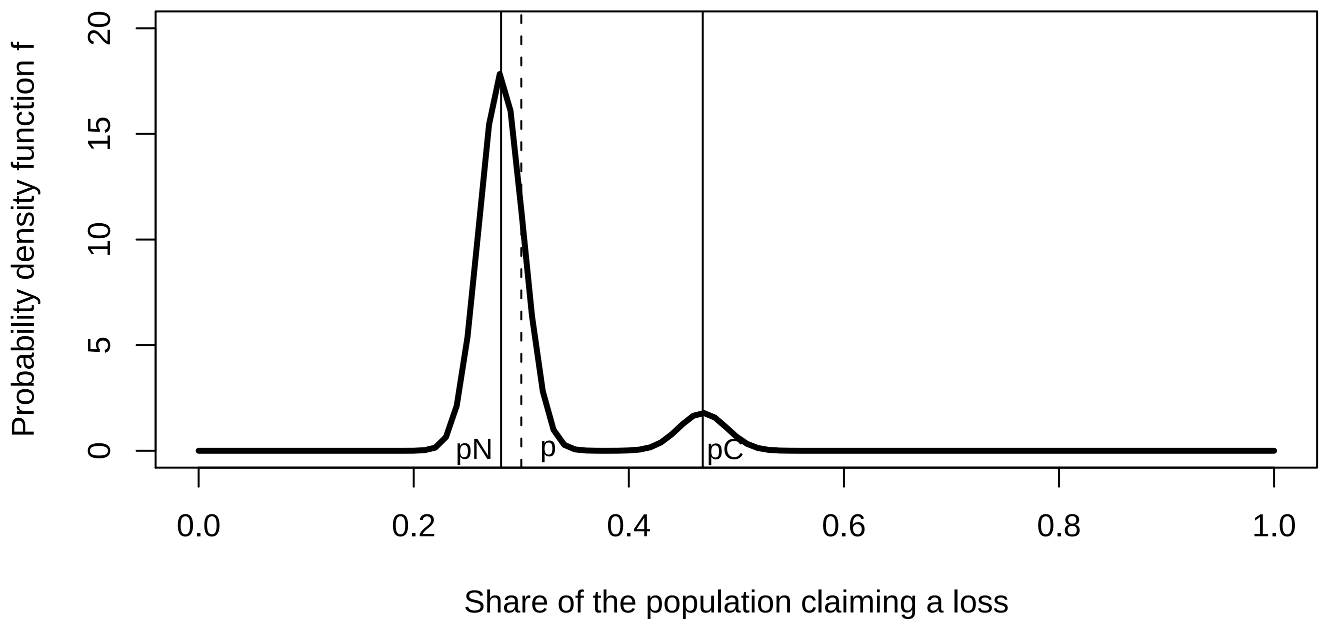
$$\begin{cases} \mathbb{P}(Y_i = 1|\Theta = \text{ Catastrophe}) = p_C \\ \mathbb{P}(Y_i = 1|\Theta = \text{ No Catastrophe}) = p_N \end{cases}$$

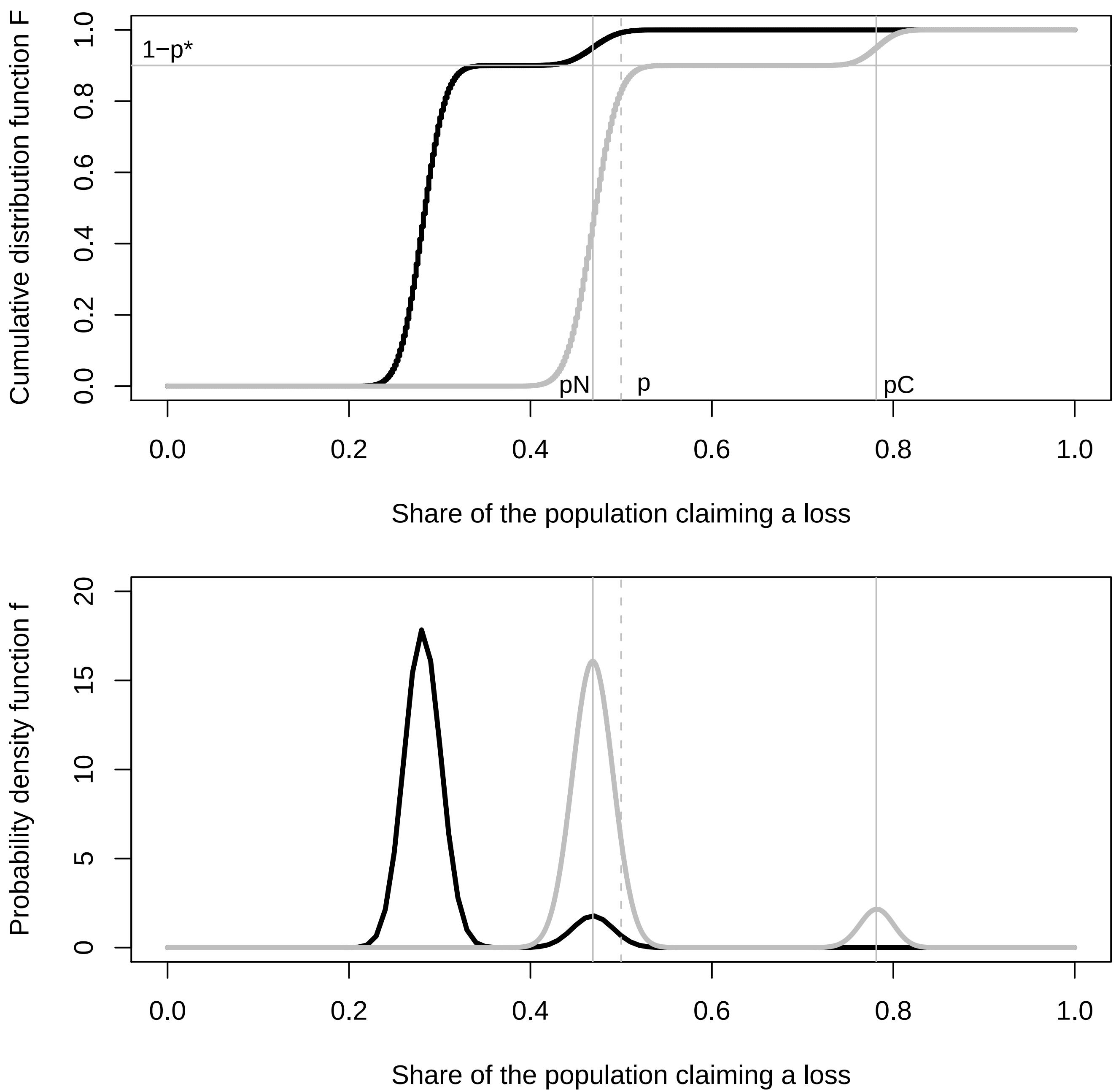
Let $p^\star = \mathbb{P}(\text{Cat})$. Then the distribution of X is

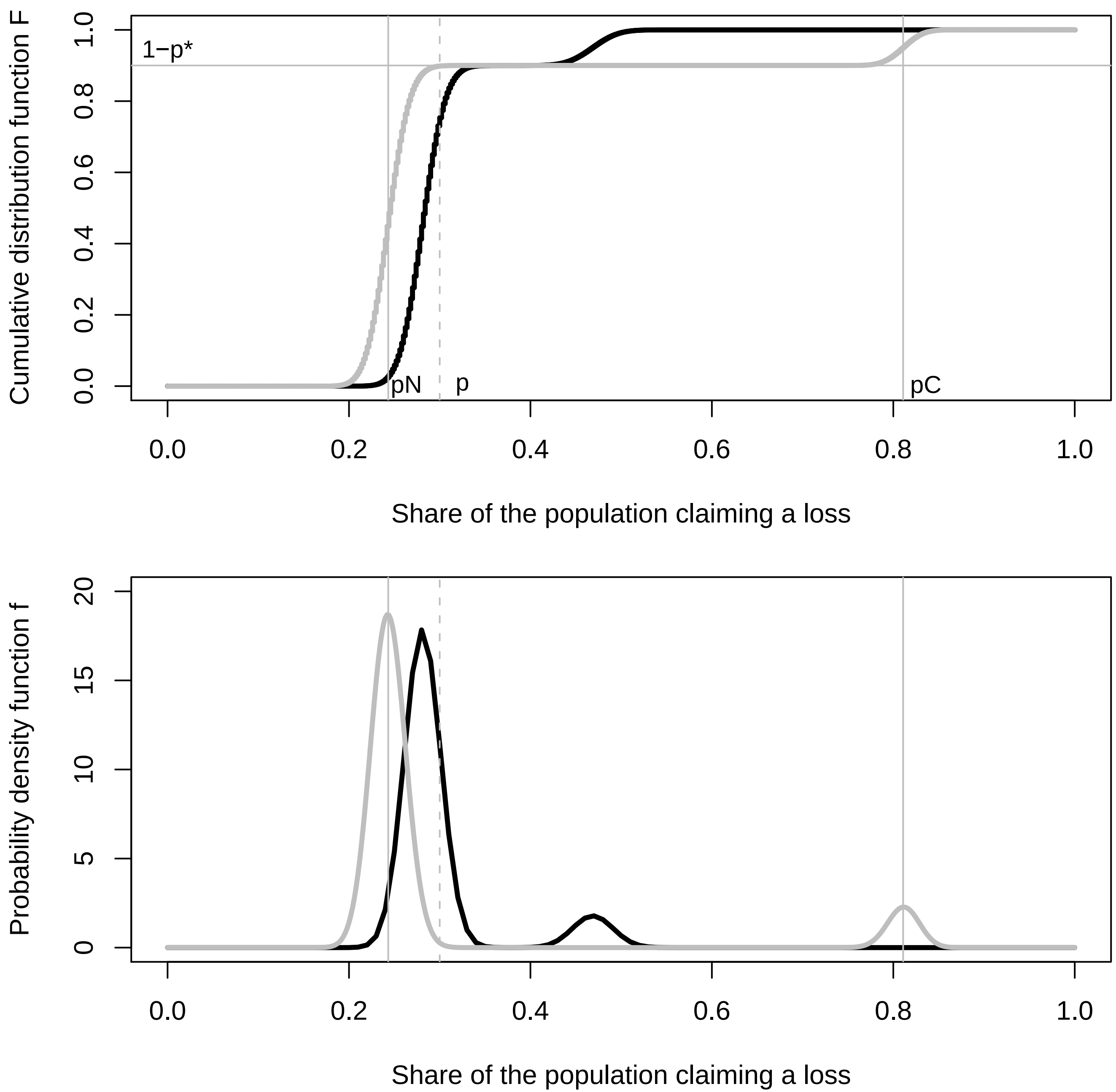
$$F(x) = \mathbb{P}(N \leq [nx]) = \mathbb{P}(N \leq k|\text{No Cat}) \times \mathbb{P}(\text{No Cat}) + \mathbb{P}(N \leq k|\text{Cat}) \times \mathbb{P}(\text{Cat}) \quad (1)$$

$$= \sum_{j=0}^{k} \binom{n}{j} \left[(p_N)^j (1 - p_N)^{n-j} (1 - p^*) + (p_C)^j (1 - p_C)^{n-j} p^*\right] \quad (2)$$


Cumulative distribution function F
0.0
0.2
0.4
0.6
0.8
1.0
1−p*
pN
p
pC
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss


Probability density function f
0
5
10
15
20
pN
p
pC
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss


Cumulative distribution function F
1−p*
pN
p
pC
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function f
0
5
10
15
20
Share of the population claiming a loss


Cumulative distribution function F
1−p*
pN
p
pC
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function f
0
5
10
15
20
Share of the population claiming a loss

4.1 Equilibriums in the EU framework

The expected profit of the insurance company is

$$\Pi(\alpha, p, \delta, c) = \int_0^{\bar{x}} [n\alpha - xnl]\, f(x)dx - [1 - F(\bar{x})]cn \tag{3}$$

Note that a premium less than the pure premium can lead to a positive expected profit.

In Rothschild & Stiglitz (QJE, 1976) a positive profit was obtained if and only if $\alpha > p \cdot l$. Here companies have limited liabilities.

Proposition1

If agents are risk adverse, for a given premium , their expected utility is always higher with government intervention.

Démonstration. Risk adverse agents look for mean preserving spread lotteries. □

Proposition2

From the expected utilities V, we obtain the following comparative static derivatives :

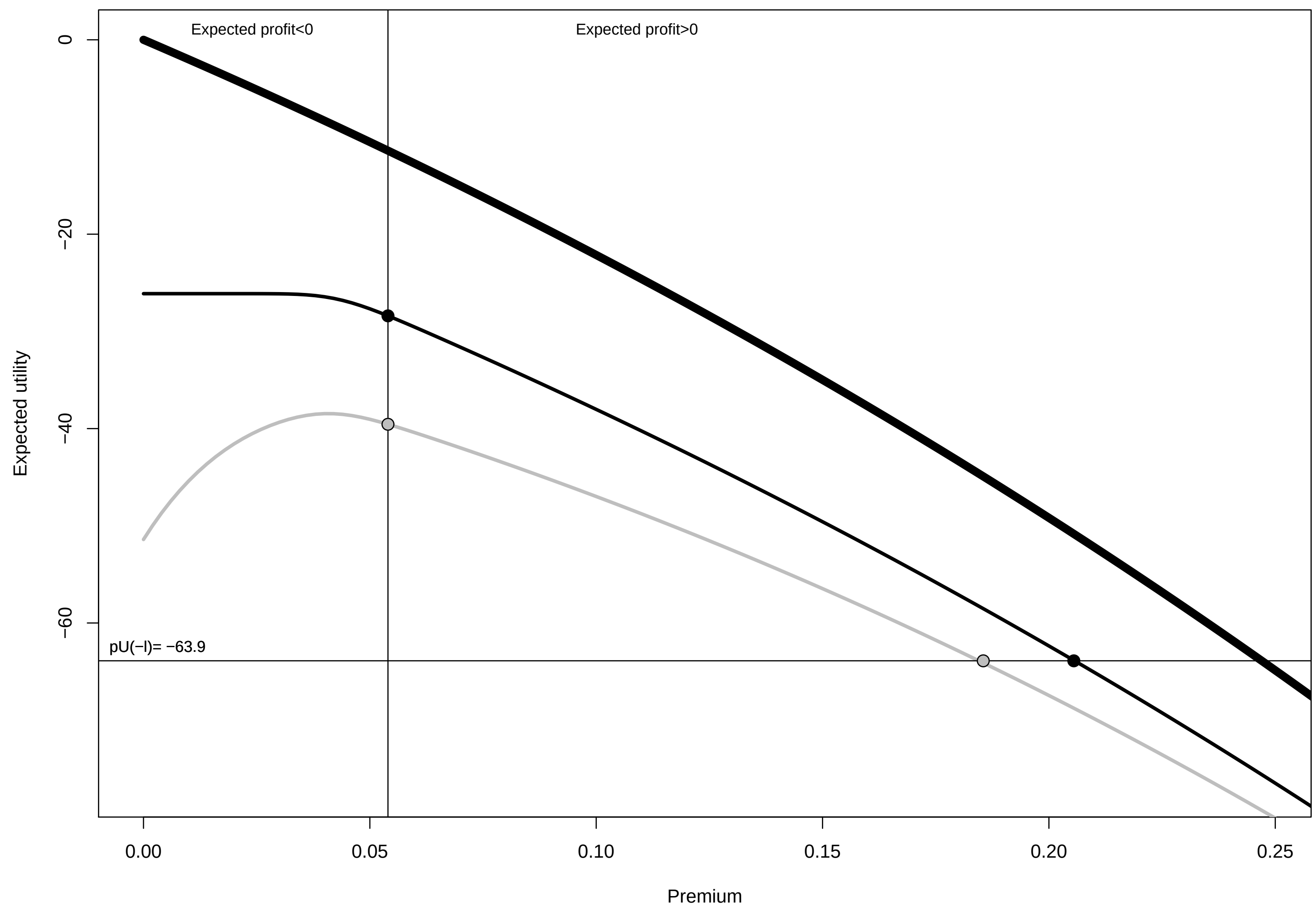
$$\frac{\partial V}{\partial \delta} < 0 \text{ for } \bar{x} > x^*, \frac{\partial V}{\partial p} < 0 \text{ for } \bar{x} > x^*, \frac{\partial V}{\partial c} > 0 \text{ for } \bar{x} \in [0;1], \frac{\partial V}{\partial \alpha} =?$$

for $\bar{x} \in [0;1]$.

Proposition3

From the equilibrium premium α^*, we obtain the following comparative static derivatives :

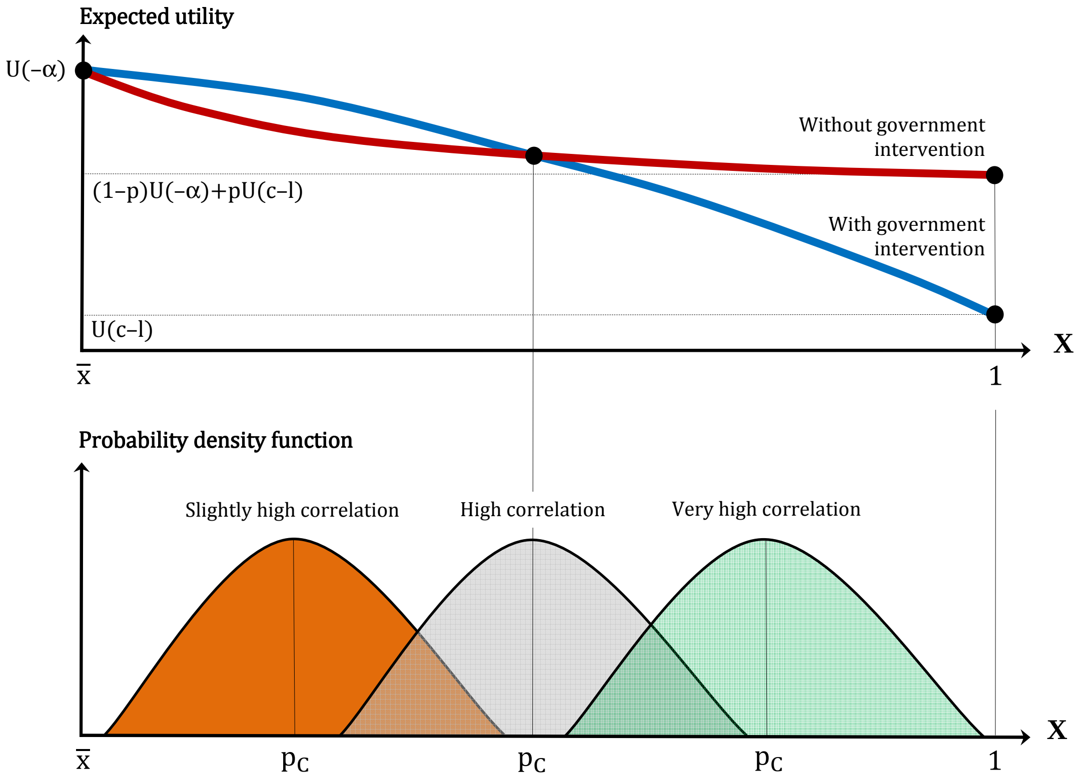
$$\frac{\partial \alpha^*}{\partial \delta} < 0 \text{ for } \bar{x} > x^*,$$
$$\frac{\partial \alpha^*}{\partial p} =? \text{ for } \bar{x} > x^*,$$
$$\frac{\partial \alpha^*}{\partial c} > 0 \text{ for } \bar{x} \in [0;1],$$

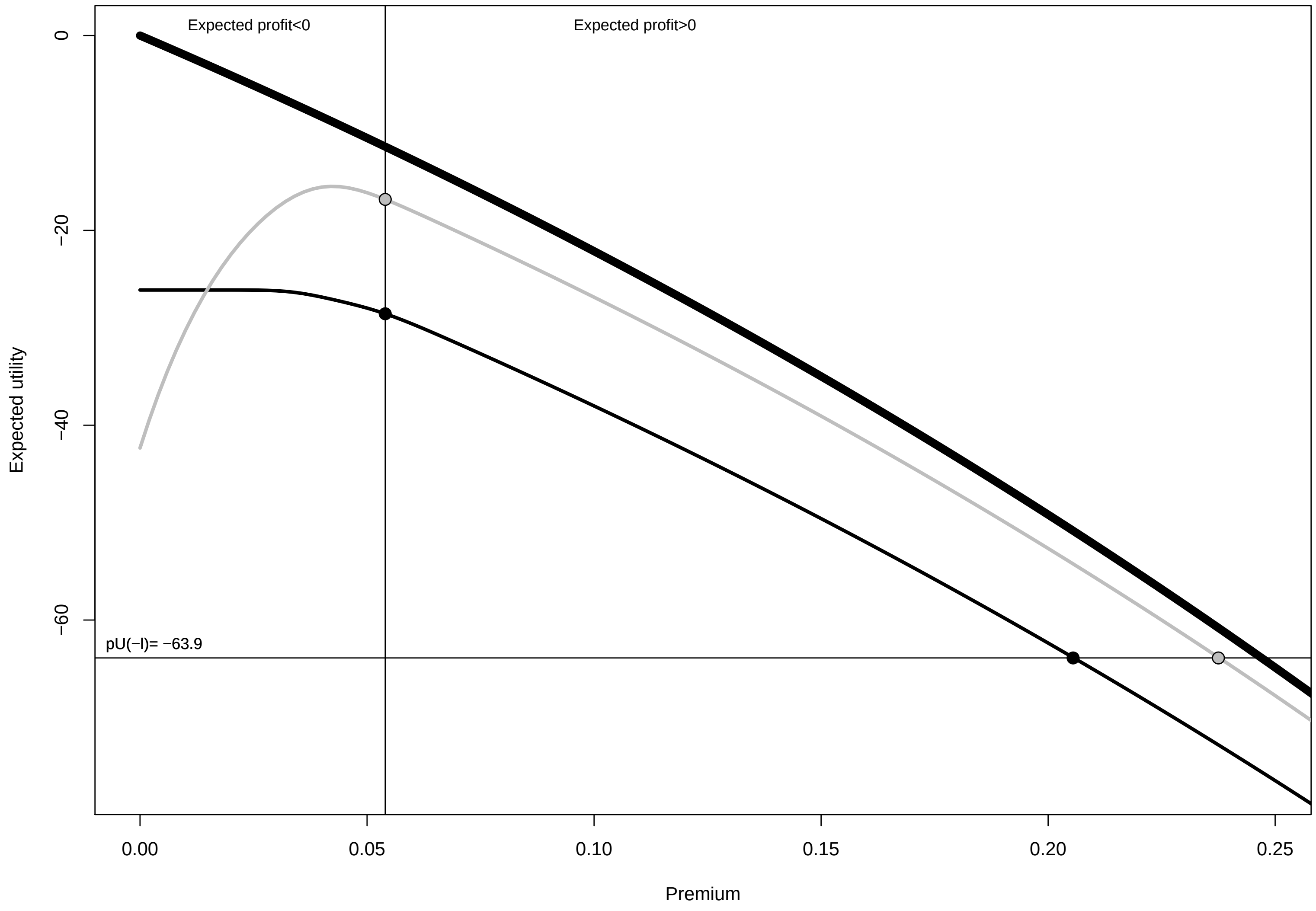

Expected profit<0
Expected profit>0
pU(−I)= −63.9
Expected utility
0
−20
−40
−60
0.00
0.05
0.10
0.15
0.20
0.25
Premium

4.2 Equilibriums in the non-EU framework

Assuming that the agents distort probabilities, they have to compare two integrals,

$$V = U(-\alpha) - \int_{\overline{x}}^{1} A_k(x) f(x) dx$$




Expected profit<0
Expected profit>0
pU(−l)= −63.9
Expected utility
0
−20
−40
−60
0.00
0.05
0.10
0.15
0.20
0.25
Premium

5 The two region model

Consider here a two-region chock model such that

- $\Theta = (0, 0)$, no catastrophe in the two regions,
- $\Theta = (1, 0)$, catastrophe in region 1 but not in region 2,
- $\Theta = (0, 1)$, catastrophe in region 2 but not in region 1,
- $\Theta = (1, 1)$, catastrophe in the two regions.

Let N_1 and N_2 denote the number of claims in the two regions, respectively, and set $N_0 = N_1 + N_2$.

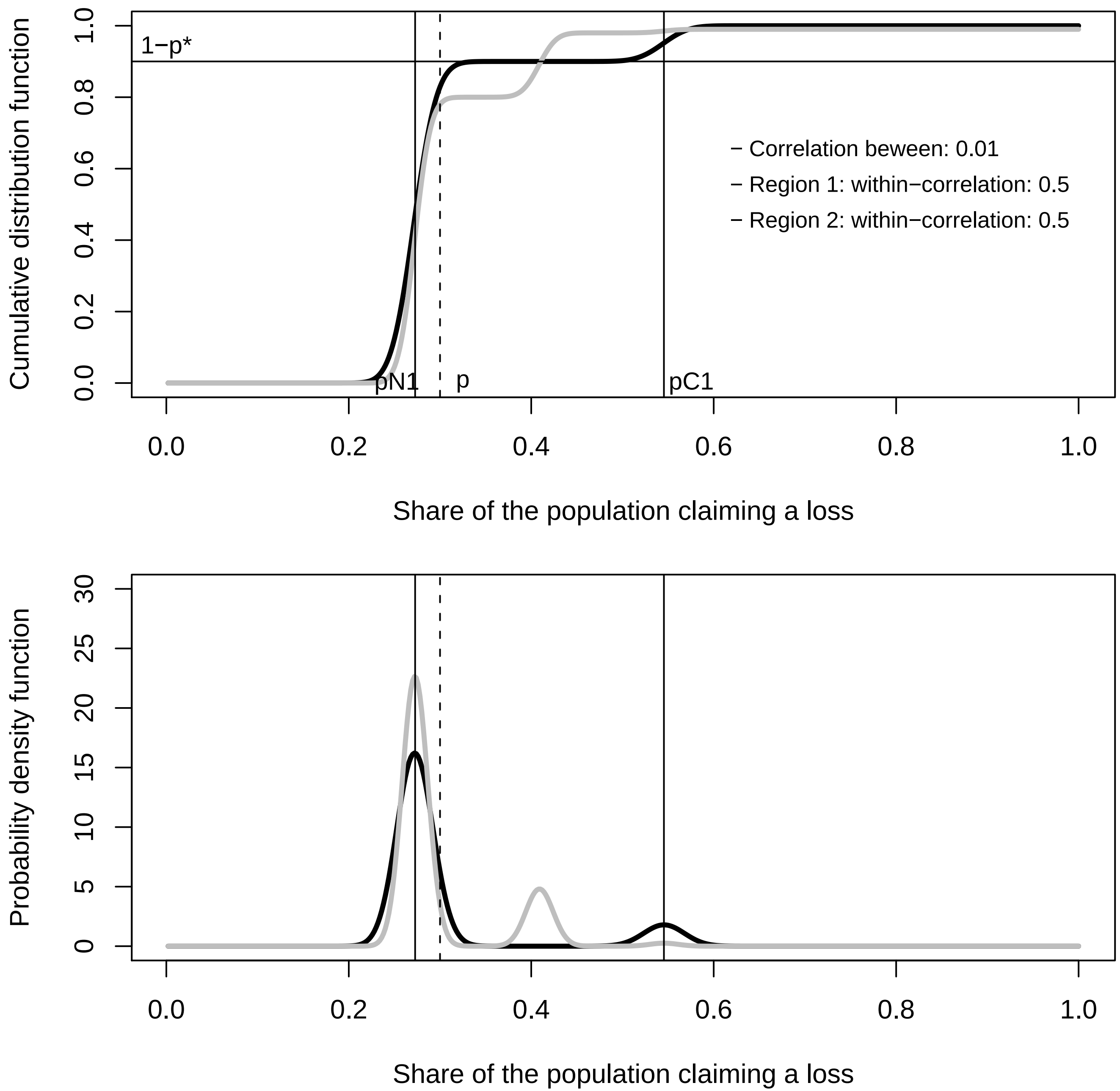
$$
\begin{aligned}
X_1 &\sim F_1(x_1|p,\delta_1) = F_1(x_1), && (4)\\
X_2 &\sim F_2(x_2|p,\delta_2) = F_2(x_2), && (5)\\
X_0 &\sim F_0(x_0|F_1,F_2,\theta) = F_0(x_0|p,\delta_1,\delta_2,\theta) = F_0(x_0), && (6)
\end{aligned}
$$

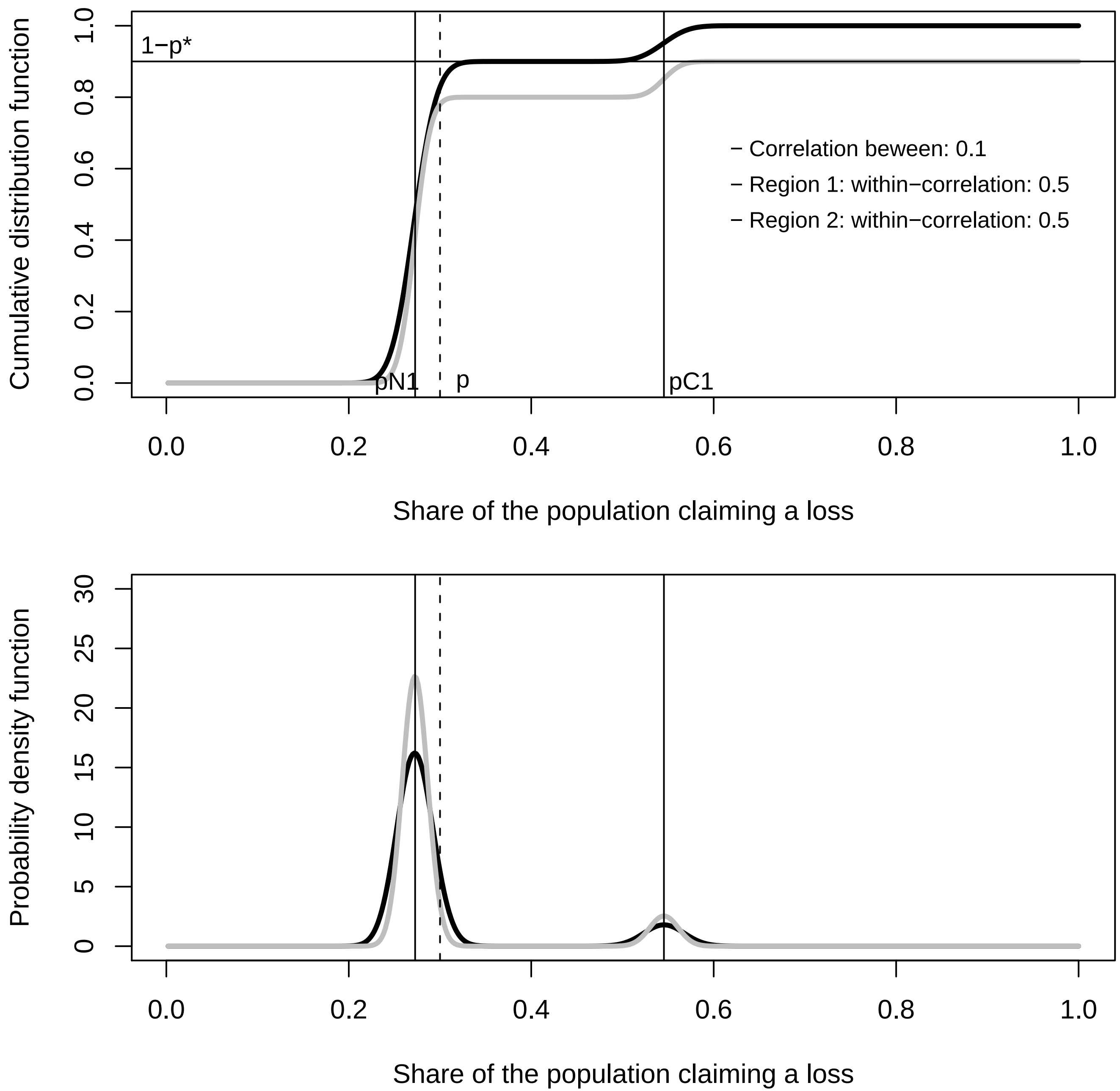
Note that there are two kinds of correlation in this model,

- a within region correlation, with coefficients δ_1 and δ_2
- a between region correlation, with coefficient δ_0

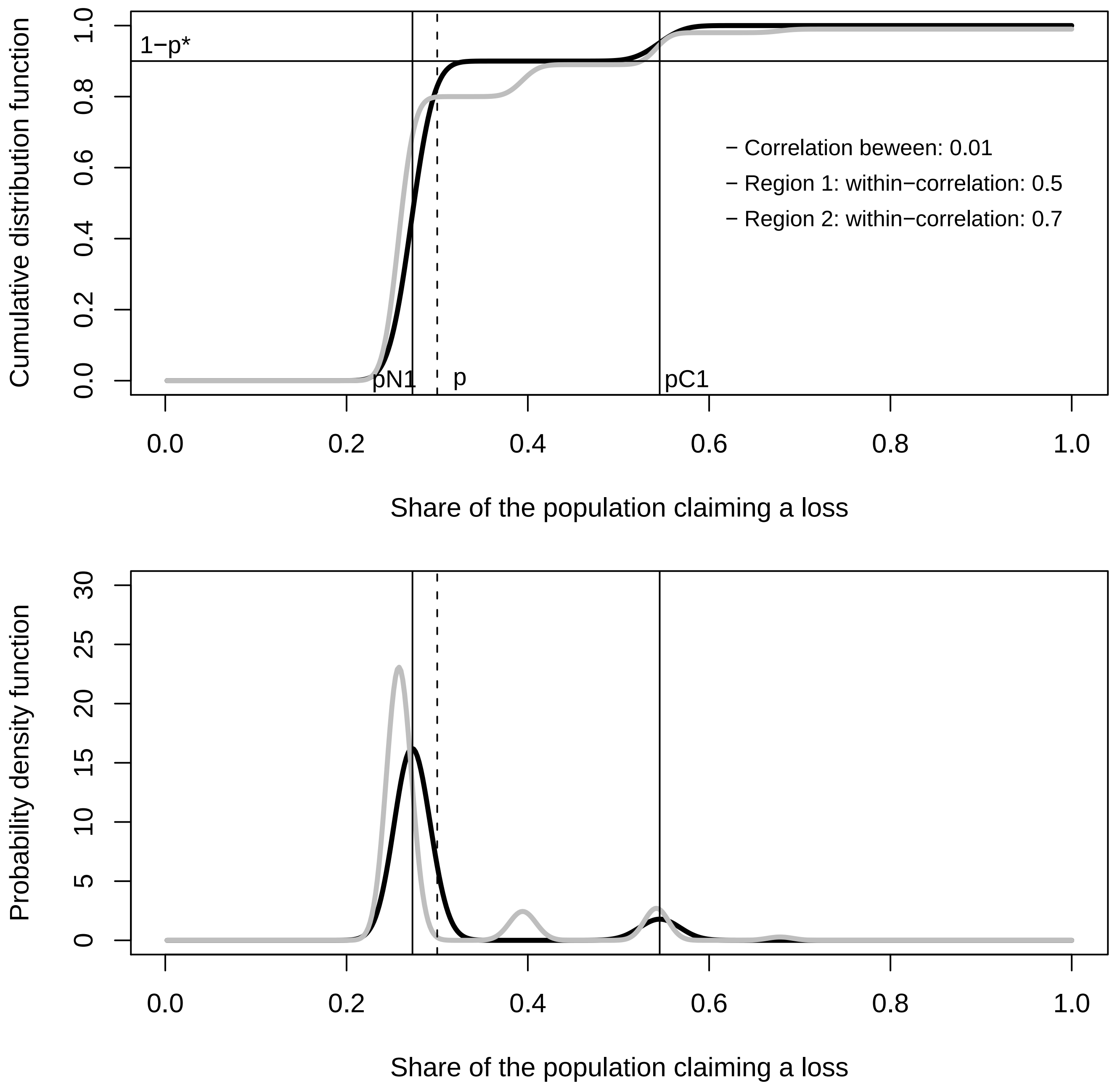
Here, $\delta_i = 1 - p_N^i/p_C^i$, where $i = 1, 2$ (Regions), while $\delta_0 \in [0, 1]$ is such that

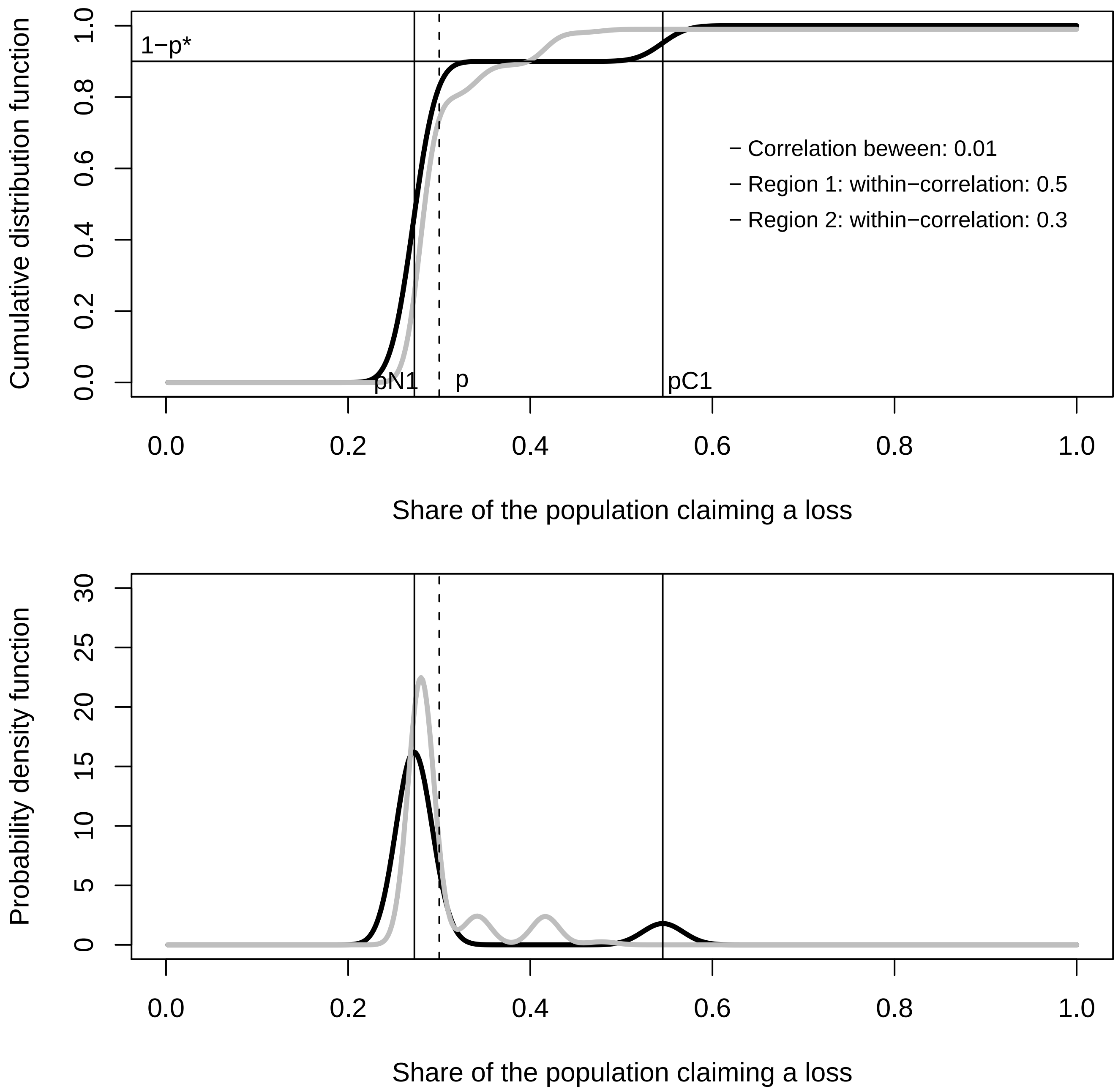
$$\mathbb{P}(\Theta = (1,1)) = \delta_0 \times \min\{\mathbb{P}(\Theta = (1,\cdot)), \mathbb{P}(\Theta = (\cdot,1))\} = \delta_0 \times \min\{p_1^\star, p_2^\star\}.$$


Cumulative distribution function
1−p*
pN1
p
pC1
− Correlation beween: 0.01
− Region 1: within−correlation: 0.5
− Region 2: within−correlation: 0.5
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function
0
5
10
15
20
25
30
Share of the population claiming a loss



Cumulative distribution function
1−p*
pN1
p
pC1
− Correlation beweeen: 0.1
− Region 1: within−correlation: 0.5
− Region 2: within−correlation: 0.5
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function
0
5
10
15
20
25
30
Share of the population claiming a loss


Cumulative distribution function
1−p*
pN1
p
pC1
− Correlation beween: 0.01
− Region 1: within−correlation: 0.5
− Region 2: within−correlation: 0.7
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss
Probability density function
0
5
10
15
20
25
30
0.0
0.2
0.4
0.6
0.8
1.0
Share of the population claiming a loss


Cumulative distribution function
0.0
0.2
0.4
0.6
0.8
1.0
1−p*
pN1
p
pC1
− Correlation beween: 0.01
− Region 1: within−correlation: 0.5
− Region 2: within−correlation: 0.3
Share of the population claiming a loss
Probability density function
0
5
10
15
20
25
30
Share of the population claiming a loss

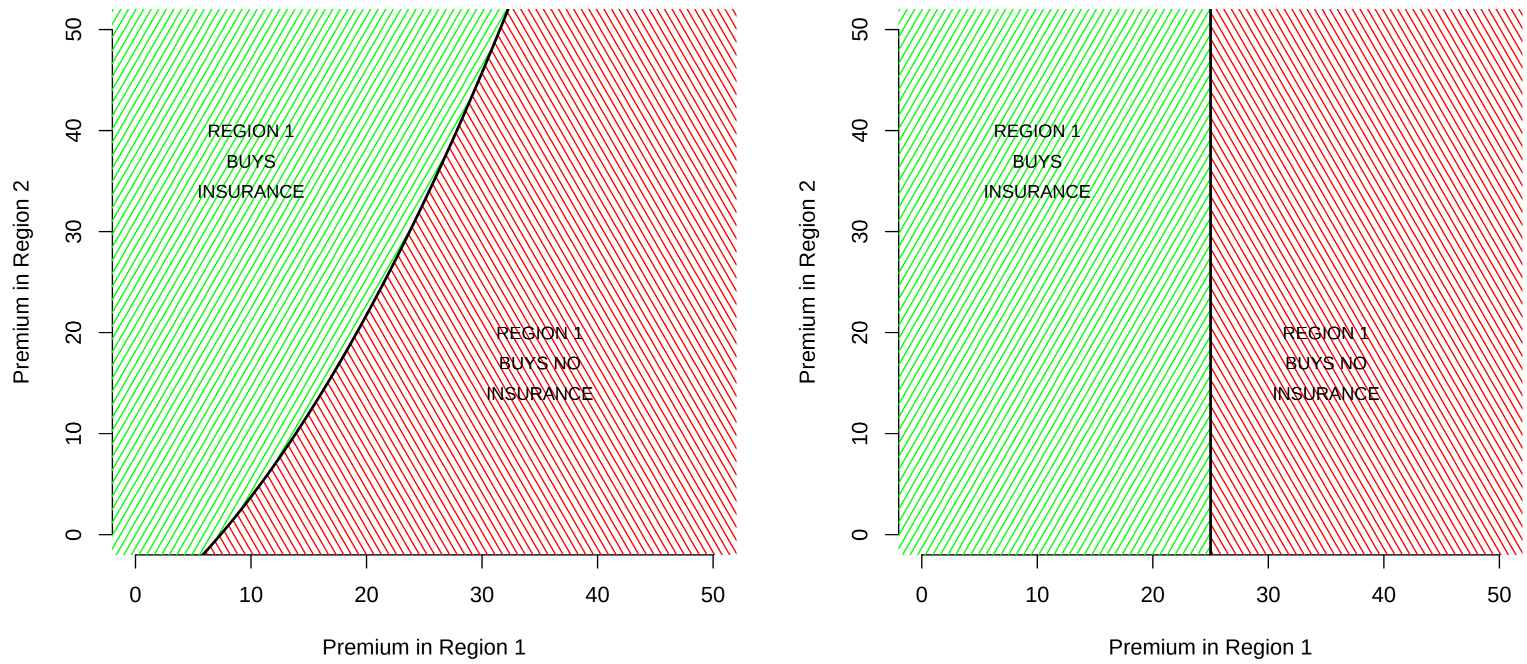
Proposition4

When both regions decide to purchase insurance, the two-region models of natural catastrophe insurance lead to the following comparative static derivatives :

$$\frac{\partial V_{i,0}}{\partial \alpha_j} > 0, \quad \frac{\partial \alpha_i^{**}}{\partial \alpha_j^{**}} > 0, \quad \text{for} \quad i = 1, 2 \quad and \quad j \neq i.$$

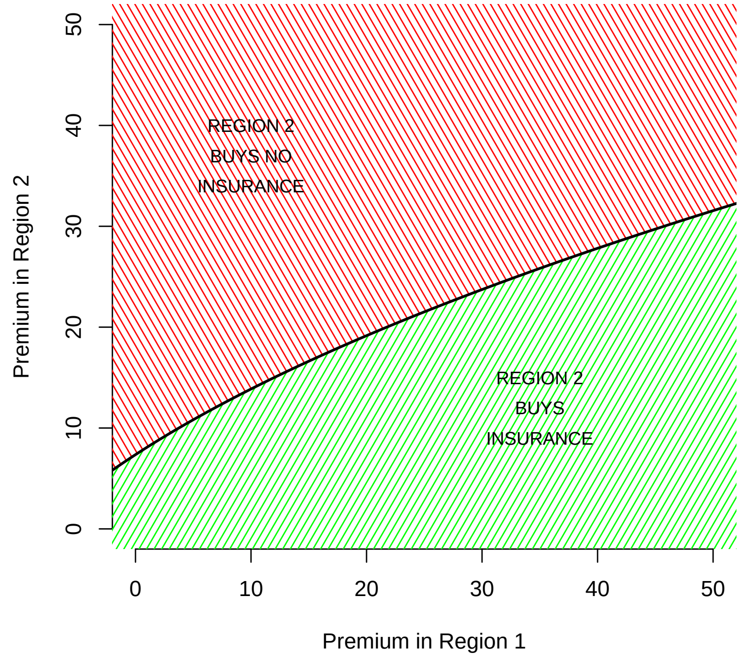
Study of the two region model

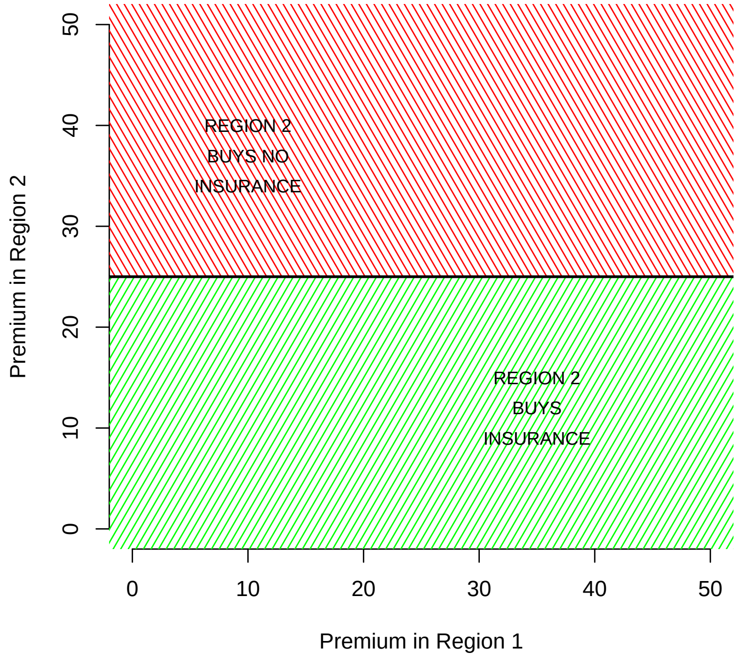
The following graphs show the decision in Region 1, given that Region 2 buy insurance (on the left) or not (on the right).



Study of the two region model

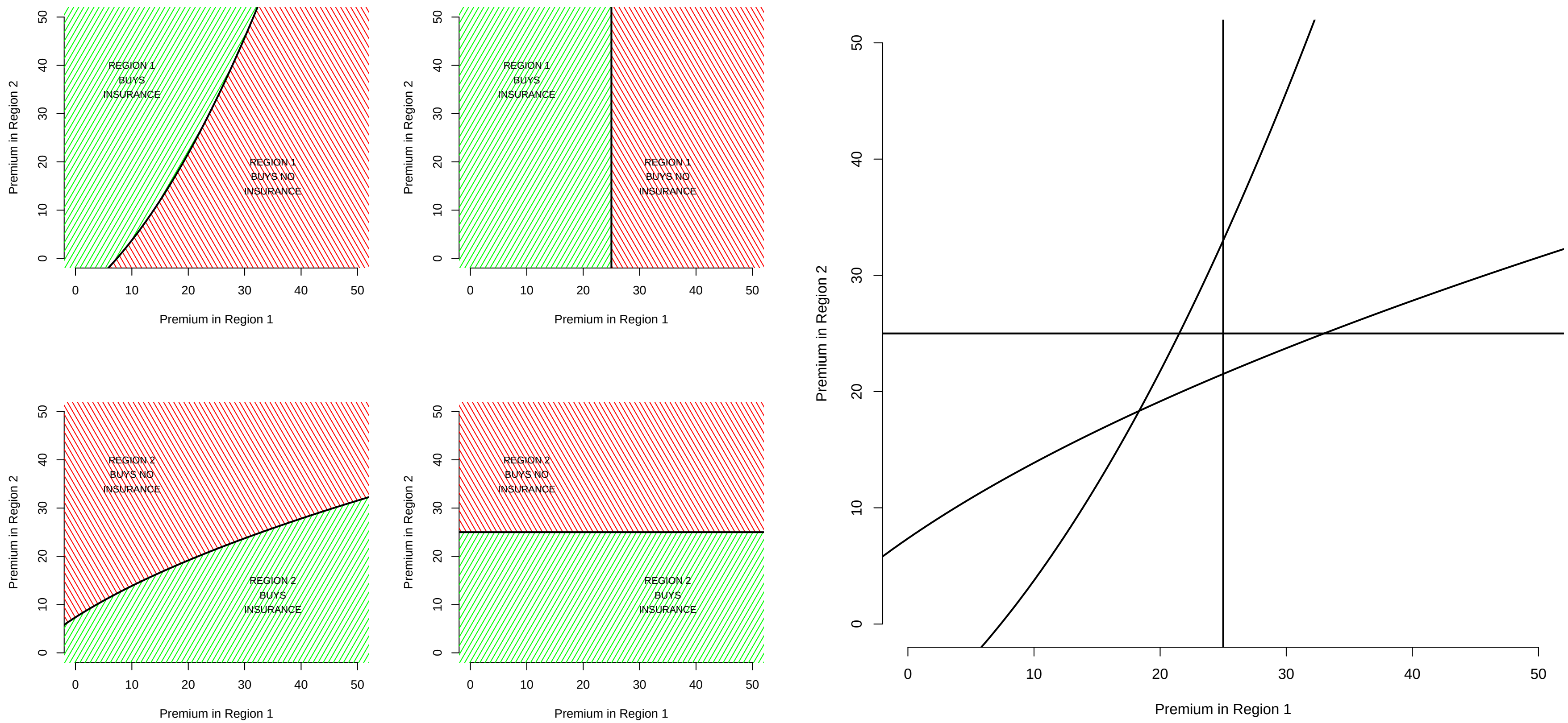
The following graphs show the decision in Region 2, given that Region 1 buy insurance (on the left) or not (on the right).





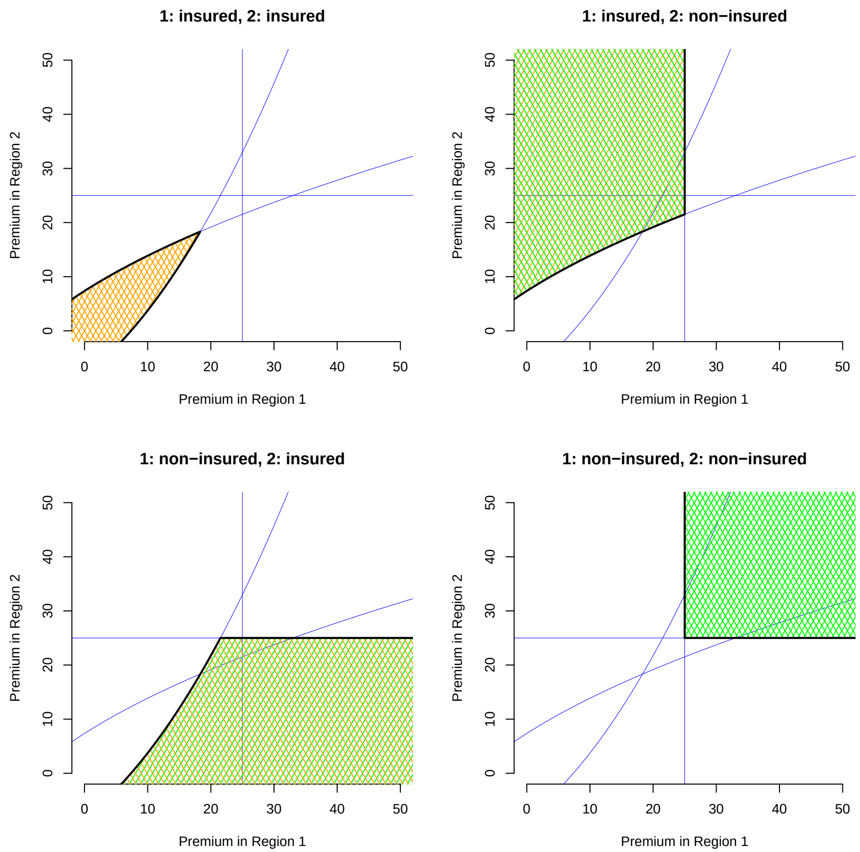
Definition1

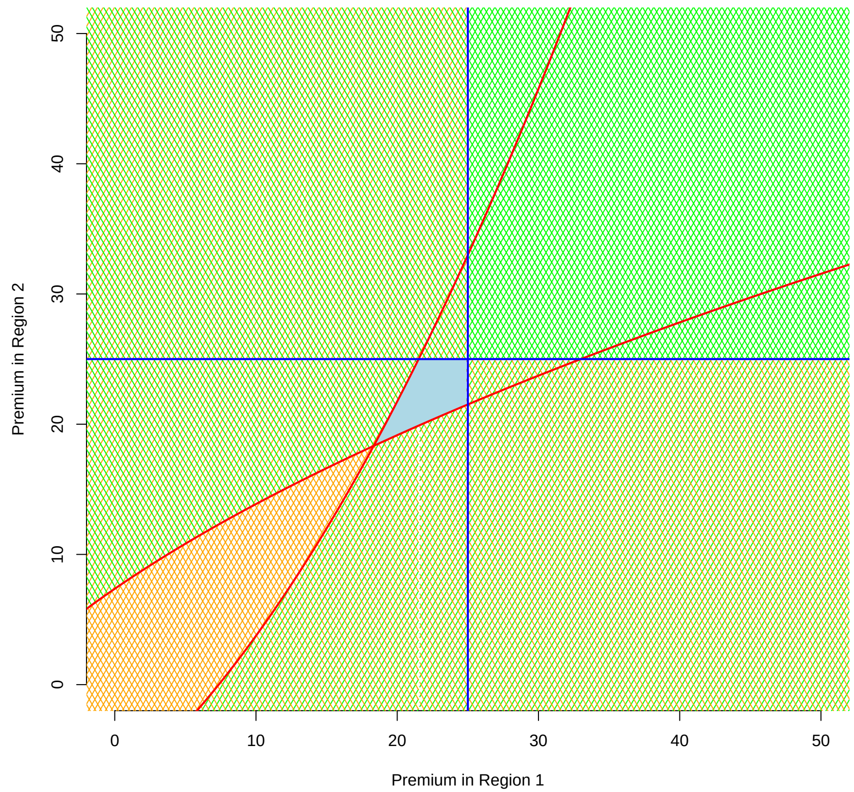
In a Nash equilibrium which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally.



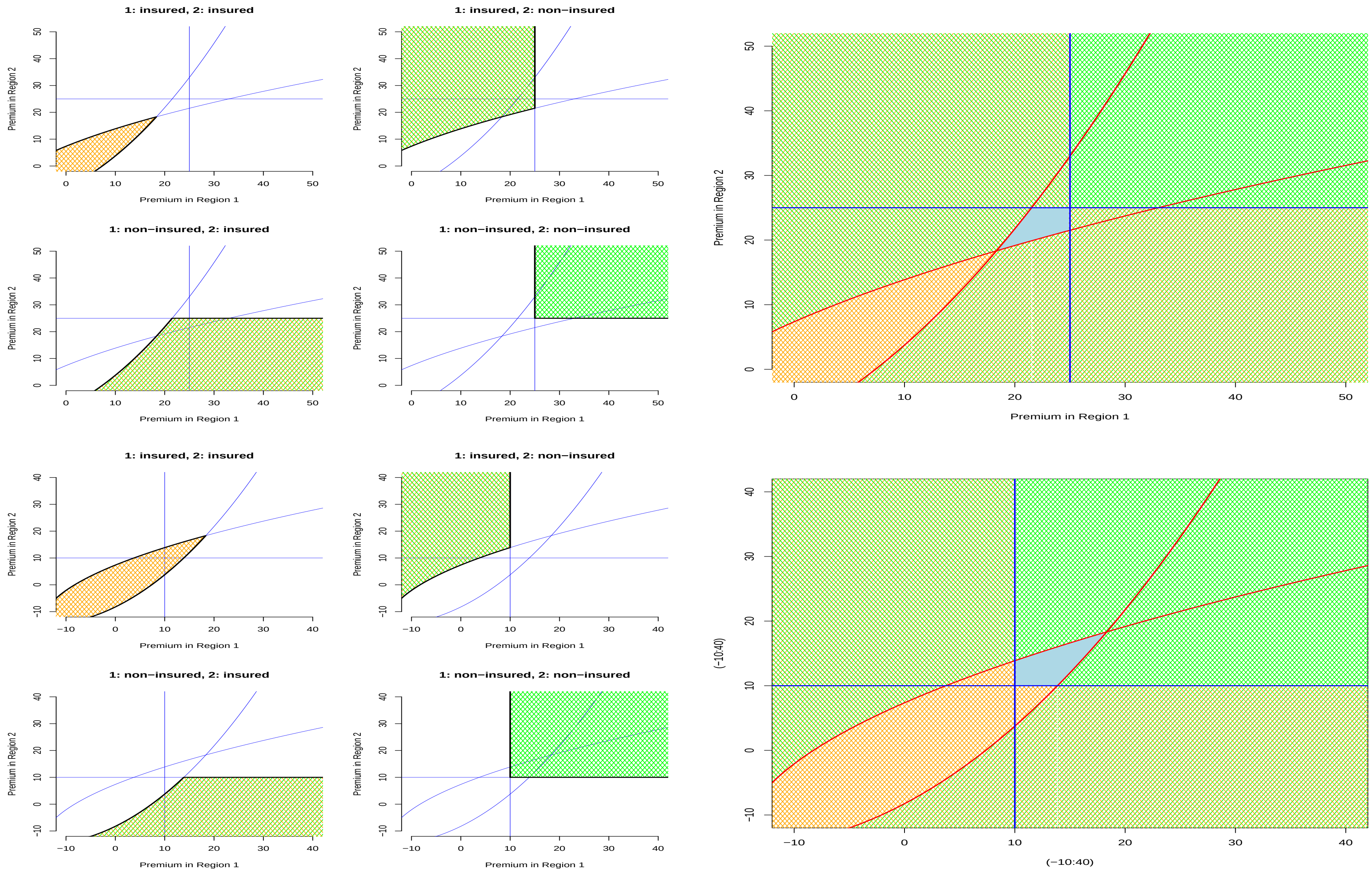
Definition2

In a **Nash equilibrium** which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally.

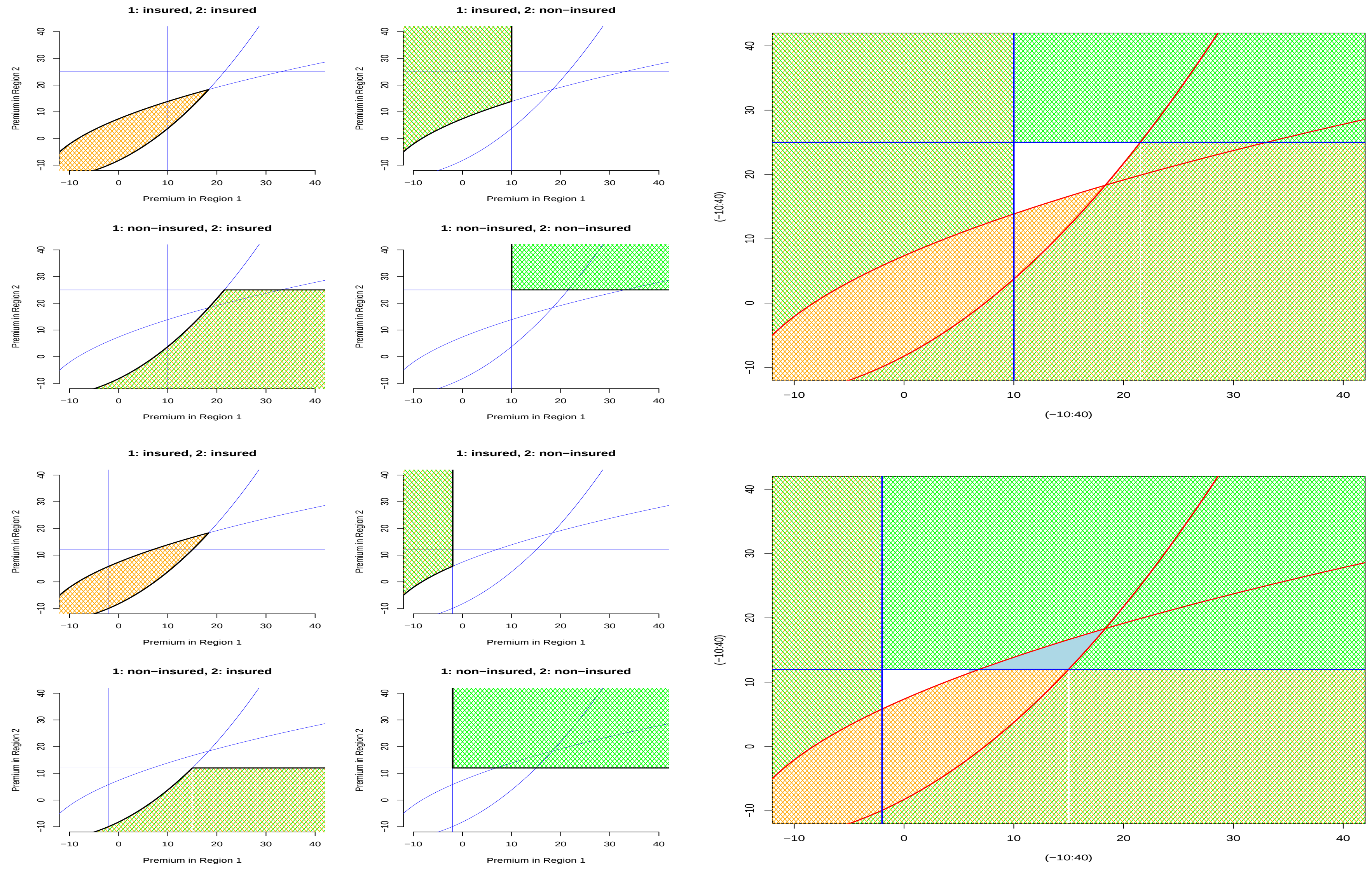




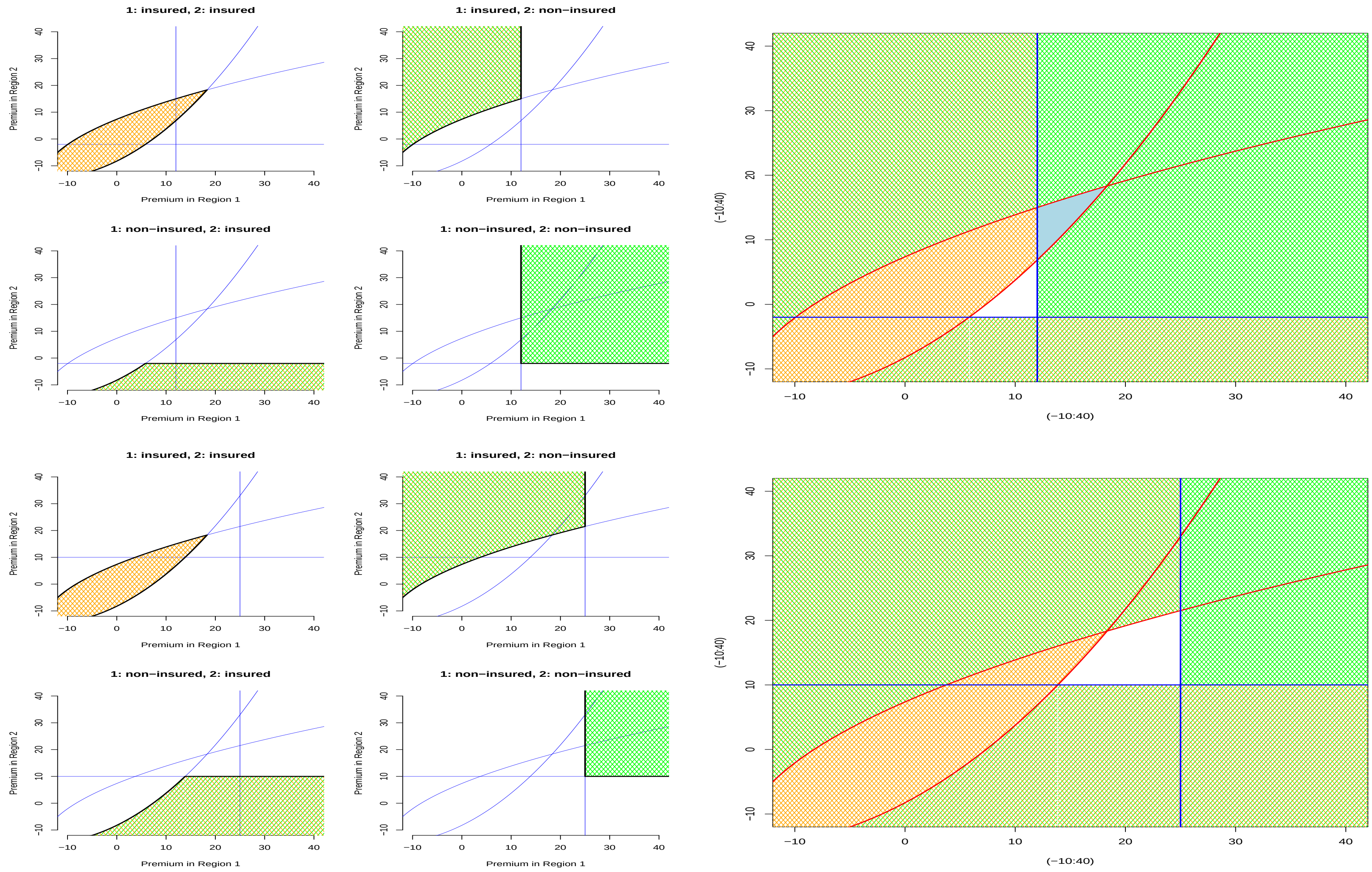
Possible Nash equilibriums



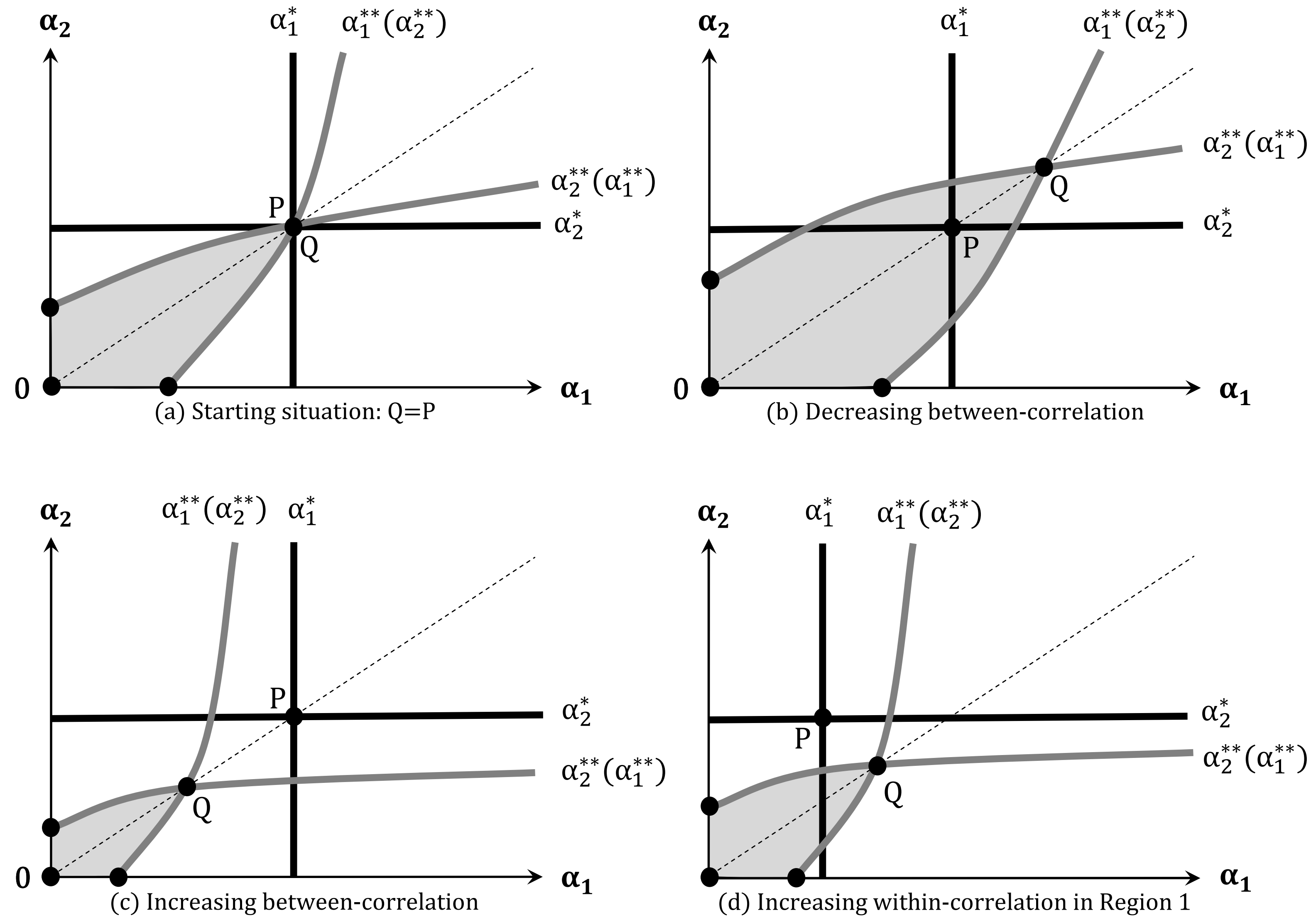
Possible Nash equilibriums



Possible Nash equilibriums



When the risks between two regions are not sufficiently independent, the pooling of the risks can lead to a Pareto improvement only if the regions have identical within-correlations, ceteris paribus. If the within-correlations are not equal, then the less correlated region needs the premium to decrease to accept the pooling of the risks.


α2
α1*
α1**(α2**)
α2**(α1**)
α2*
P
Q
0
α1
(a) Starting situation: Q=P
(b) Decreasing between-correlation
(c) Increasing between-correlation
(d) Increasing within-correlation in Region 1