Modeling dynamic incentives
an application to basketball

Arthur Charpentier[1,2,3], Nathalie Colombier[2] & Romuald Élie[4,5]

[1] UQAM [2] Univ. Rennes 1 - CREM [3] École Polytechnique [4] Univ. Paris Dauphine [5] CREST

Journées SCSE, Mai 2012

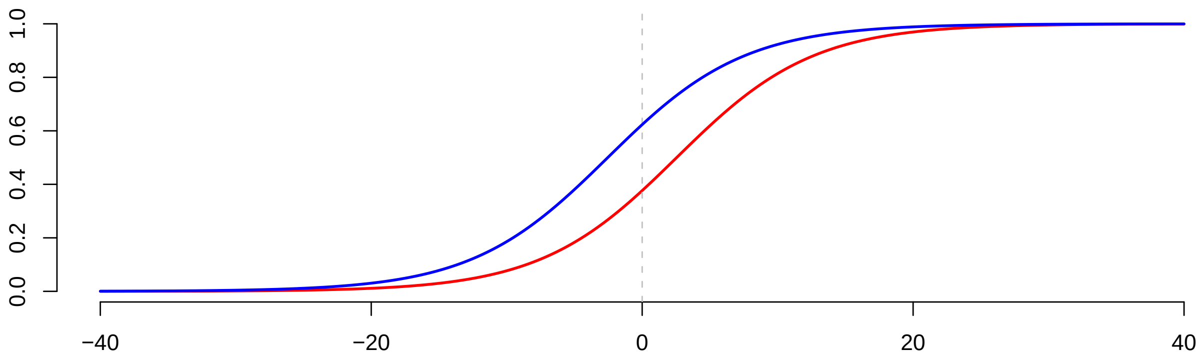
Why such an interest in basketball ?

Recent preprint '*Can Losing Lead to Winning ?*' by Berger and Pope (2009). See also New York Times, Boston Globe, Wall Street Journal, ESPN.com, Freakonomics, etc.

Focus on winning probability in basketball games,

$$\text{win}_i = \alpha + \beta(\text{losing at half time})_i + \delta(\text{score difference at half time})_i + \gamma \boldsymbol{X}_i + \varepsilon_i$$

$\boldsymbol{X}_i$ is a matrix of *control variables* for game i



Modeling dynamic incentives ?

Dataset on college basketball match, but the original dataset had much more information : score difference from halftime until the end (per minute).

$\Longrightarrow$ a dynamic model to understand *when* losing lead to losing

(or winning lead to winning).

Talk on '*Point Record Incentives, Moral Hazard and Dynamic Data*' by Dionne, Pinquet, Maurice & Vanasse (2011)

Study on incentive mechanisms for road safety, with time-dependent disutility of effort

Agenda of the talk

- **From basketball to labor economics**
- **An optimal effort control problem**
  - A simple control problem
  - Nash equilibrium of a stochastic game
  - Numerical computations
- **Understanding the dynamics : modeling processes**
  - The score process
  - The score difference process
  - A proxy for the effort process
- **Modeling winning probabilities**

Incentives and tournament in labor economics

The pay schemes : Flat wage pay *versus* Piece rate or rank-order tournament (relative performance evaluation).

Impact of relative performance evaluation (Lazear, 1989) :

- motivate employees to work harder
- demoralizing and create excessively competitive workplace

Incentives and tournament in labor economics

For a given pay scheme : how intensively should the organization provide his employees with information about their relative performance ?

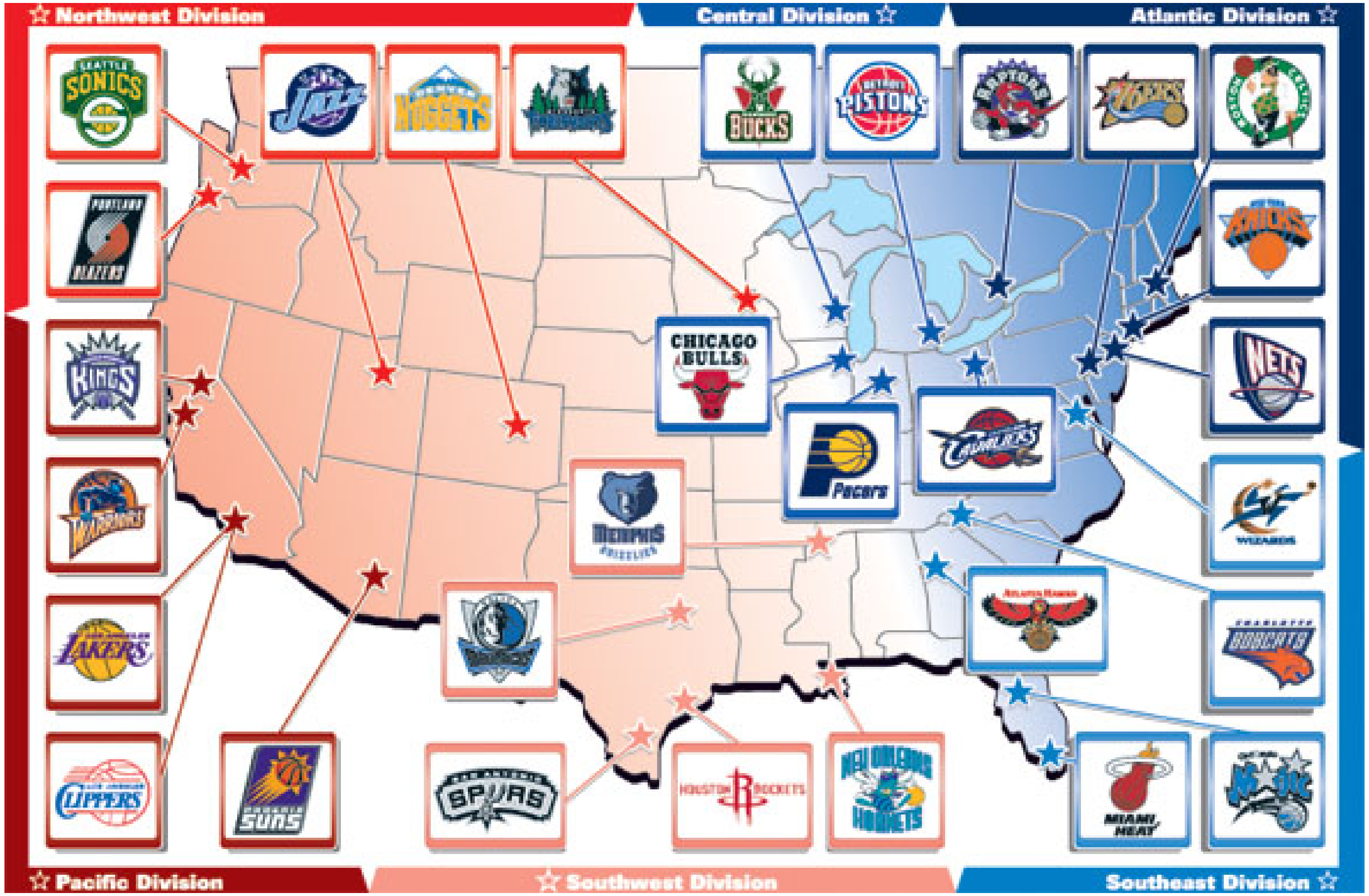
- *An employee who is informed he is an underdog*
  - may be discouraged and lower his performance
  - works harder to preserve to avoid shame
- *A frontrunner who learns that he is well ahead*
  - may think that he can afford to slack
  - becomes more enthusiastic and increases his effort

Incentives and tournament in labor economics

⇒ impact on overall perfomance ?

- **Theoritical models** conclude to a positive impact (Lizzeri, Meyer and Persico, 2002 ; Ederer, 2004)
- **Empirical literature** :
  - if payment is independant of the other's performance : positive impact to observe each other's effort (Kandel and Lazear, 1992).
  - in relative performance (both tournament and piece rate) : does not lead frontrunners to slack off but significantly reduces the performance of underdogs (quantity vs. quality) (Eriksson, Poulsen and Villeval, 2009).

The dataset for 2008/2009 NBA match

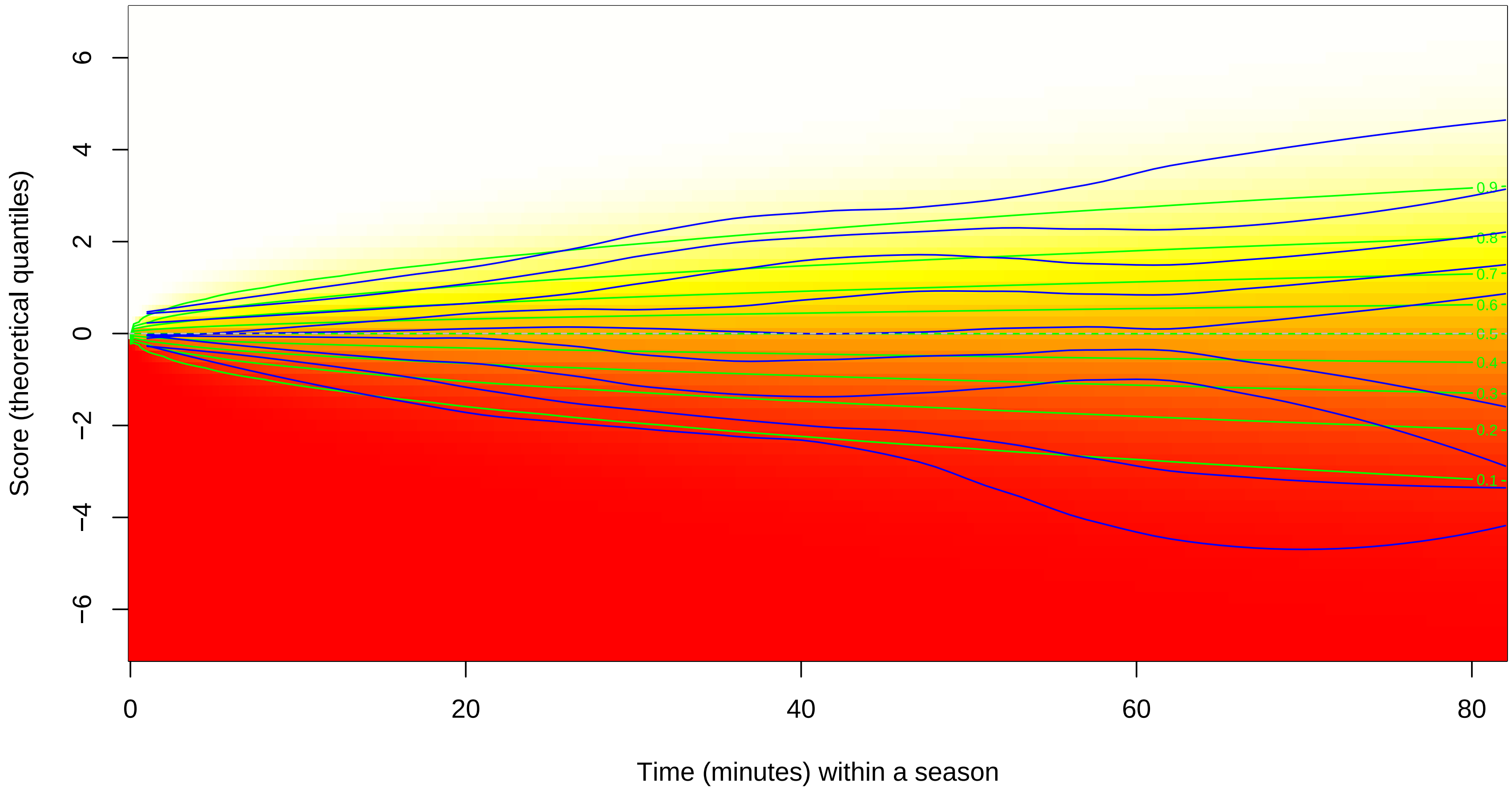


The dataset for 2008/2009 NBA match

Atlantic Division	W	L	Northwest Division	W	L
Boston Celtics	62	20	Denver Nuggets	54	28
Philadelphia 76ers	41	41	Portland Trail Blazers	54	28
New Jersey Nets	34	48	Utah Jazz	48	34
Toronto Raptors	33	49	Minnesota Timberwolves	24	58
New York Knicks	32	50	Oklahoma City Thunder	23	59
DCentral Division	W	L	Pacific Division	W	L
Cleveland Cavaliers	66	16	Los Angeles Lakers	65	17
Chicago Bulls	41	41	Phoenix Suns	46	36
Detroit Pistons	39	43	Golden State Warriors	29	53
Indiana Pacers	36	46	Los Angeles Clippers	19	63
Milwaukee Bucks	34	48	Sacramento Kings	17	65
SoutheastDivision	W	L	Southwest Division	W	L
Orlando Magic	59	23	San Antonio Spurs	54	28
Atlanta Hawks	47	35	Houston Rockets	53	29
Miami Heat	43	39	Dallas Mavericks	50	32
Charlotte Bobcats	35	47	New Orleans Hornets	49	33
Washington Wizards	19	63	Memphis Grizzlies	24	58

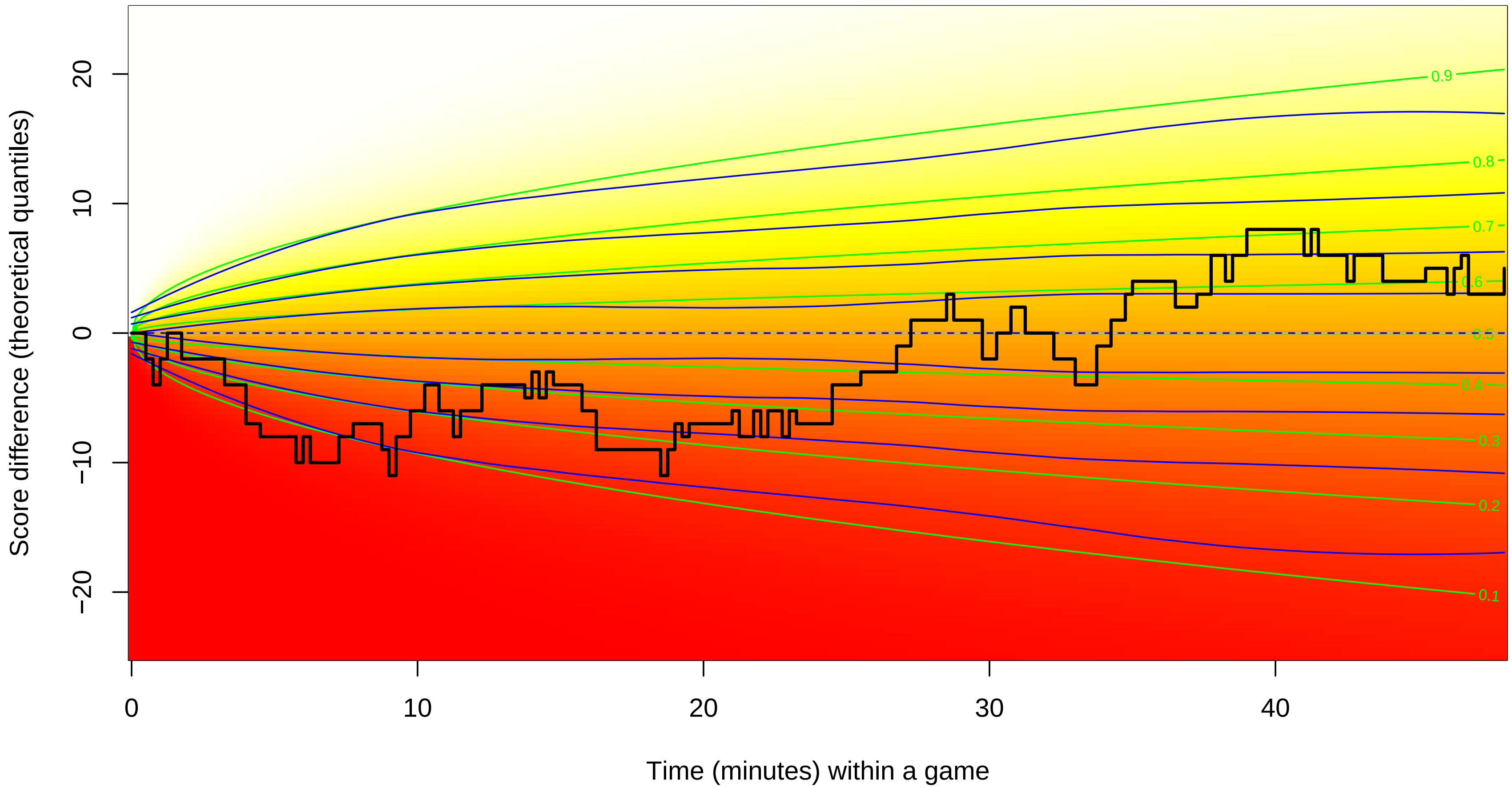
A Brownian process to model the season (LT) ?

Marginal confidence bands of a brownian motion (——) versus empirical score (smoothed version ——) $\Longrightarrow$ increments with positive dependence



A Brownian process to model the score difference (ST) ?

Marginal confidence bands of a brownian motion (——) versus empirical score difference (smoothed version ——) $\Longrightarrow$ increments with negative dependence



Introducing the effort as a control process

There are two players (teams), 1 and 2, playing a **game** over a period $[0,T]$. Let (S_t) denote the score difference (in favor of team 1 w.r.t. team 2)

- team 1 : $\displaystyle\max_{(u_1)\in\mathcal{U}_1}\left\{\mathbb{E}\left([\alpha_1\mathbf{1}(S_T>0)]+\int_\tau^T e^{-\delta_1 t}L_1(\alpha_1-u_{1,t})\right)dt\right\}$
- team 2 : $\displaystyle\max_{(u_2)\in\mathcal{U}_2}\left\{\mathbb{E}\left([\alpha_2\mathbf{1}(S_T<0)]+\int_\tau^T e^{-\delta_2 t}L_2(\alpha_2-u_{2,t})\right)dt\right\}$

where (S_t) is a **stochastic** process

Introducing the effort as a control process

There are two players (teams), 1 and 2, playing a **game** over a period $[0,T]$. Let (S_t) denote the score difference (in favor of team 1 w.r.t. team 2)

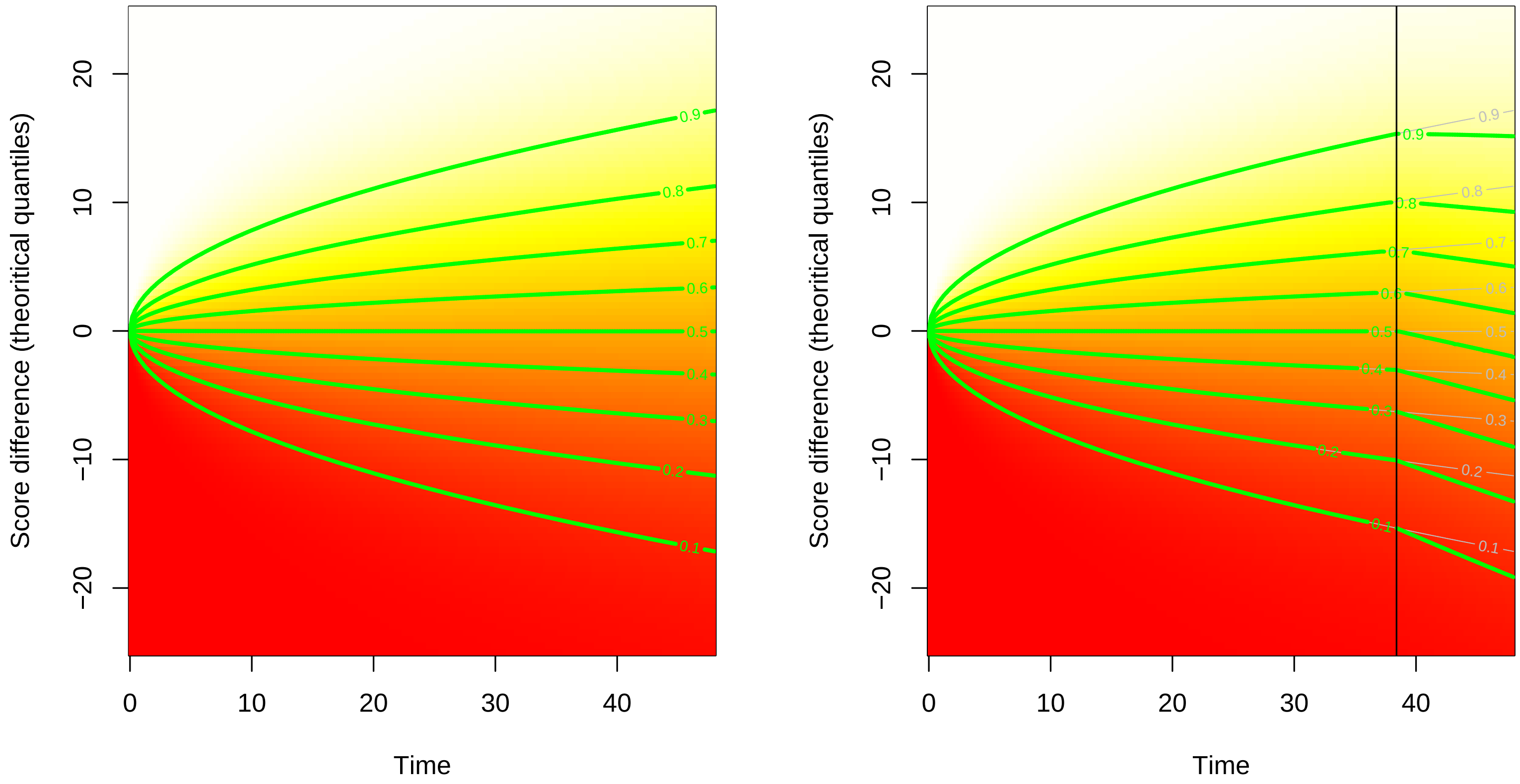
- team 1 : $\displaystyle\max_{(u_1)\in\mathcal{U}_1}\left\{\mathbb{E}\left([\alpha_1\mathbf{1}(S_T>0)]+\int_\tau^T e^{-\delta_1 t}L_1(\alpha_1-u_{1,t})\right)dt\right\}$
- team 2 : $\displaystyle\max_{(u_2)\in\mathcal{U}_2}\left\{\mathbb{E}\left([\alpha_2\mathbf{1}(S_T<0)]+\int_\tau^T e^{-\delta_2 t}L_2(\alpha_2-u_{2,t})\right)dt\right\}$

where (S_t) is a **stochastic** process driven by

$$dS_t=[u_1(S_t)-u_2(S_t)]dt+\sigma dW_t \text{ on } [0,T].$$

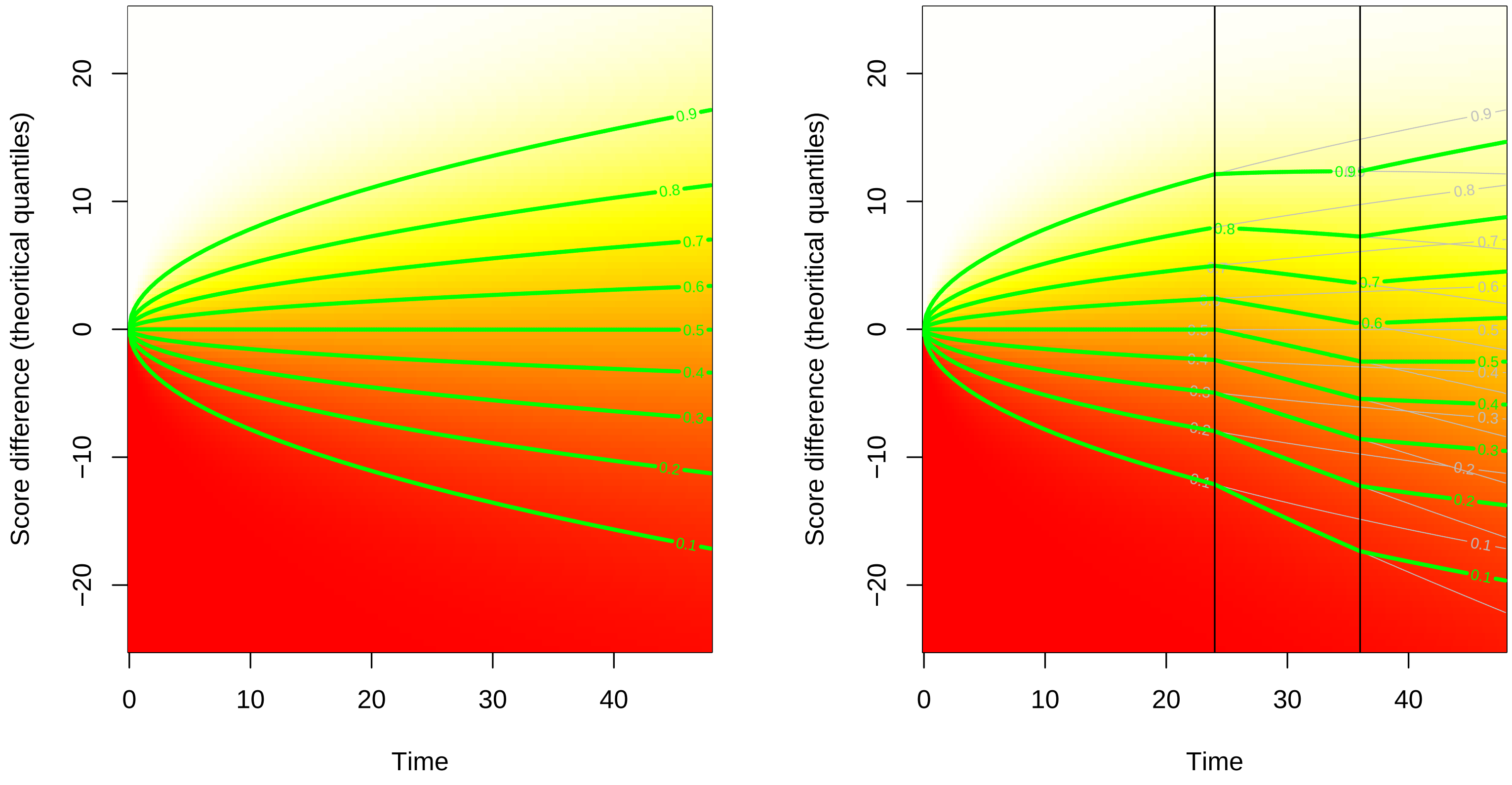
Introducing the effort as a control process

Assume for instance that the first team changed its effort after 38 minutes,



Introducing the effort as a control process

... or changed its effort after 24 minutes, and again after 36 minutes,



An optimal control stochastic game

There are two players (teams), 1 and 2, playing a **game** over a period $[0,T]$. Let (S_t) denote the score difference (in favor of team 1 w.r.t. team 2)

- team 1 : $u^\star_{1,\tau} \in \underset{(u_1)\in\mathcal{U}_1}{\text{argmax}} \left\{ \mathbb{E}\left([\alpha_1 \mathbf{1}(S_T > 0)] + \int_\tau^T e^{-\delta_1 t} L_1(\alpha_1 - u^\star_{1,t}(S_t)) \right) dt \right\}$
- team 2 : $u^\star_{2,\tau} \in \underset{(u_2)\in\mathcal{U}_2}{\text{argmax}} \left\{ \mathbb{E}\left([\alpha_2 \mathbf{1}(S_T < 0)] + \int_\tau^T e^{-\delta_2 t} L_2(\alpha_2 - u^\star_{2,t}(S_t)) \right) dt \right\}$

where (S_t) is a **stochastic** process driven by

$$dS_t = [u^\star_{1,t}(S_t) - u^\star_{2,t}(S_t)]dt + \sigma dW_t \text{ on } [0,T].$$

$\Longrightarrow$ non-cooperative stochastic (dynamic) game with 2 players and non-null sum

An optimal control problem

Consider now not a game, but a standard optimal control problem, where an agent faces the optimization program

$$\max_{(\gamma_t)_{t\in[\tau,T]}}\left\{\mathbb{E}\left(\mathbf{1}(S_T>0)+\int_\tau^T e^{-\delta t}L(\alpha-u_t)dt\right)\right\},$$

with

$$dS_t = u_t(S_t)dt+\sigma dW_t$$

where L is an increasing convex utility function, with $\alpha>0$, and $\delta>0$.

Consider a two-value effort model,

- if $u_t=0$, there is fixed utility $u(\alpha)$
- if $u_t=u>0$, there an **decrease** of utility $L(\alpha-u)<L(\alpha)$, but also an **increase** of $\mathbb{P}(S_T>0)$ since the 'Brownian process' now has a positive drift.

When should a team stop playing (with high effort) ?

The team starts playing with a high effort (u), and then, stop effort at some time τ : utility gains exceed changes in the probability to win, i.e.

$$\int_\tau^T e^{-\delta t} L(\alpha - u)dt + \mathbb{P}(S_T > 0 | S_\tau, \text{ negative drift on } [\tau, T])$$
$$> \int_\tau^T e^{-\delta t} L(\alpha)dt + \mathbb{P}(S_T > 0 | S_\tau, \text{ no drift on } [\tau, T])$$

Recall that, if $Z = S_T - S_\tau$

$$\mathbb{P}(S_T > 0 | S_\tau = d, \text{ no drift on } [\tau, T]) = \mathbb{P}(Z > -d | Z \sim \mathcal{N}(0, \sigma\sqrt{T-\tau}))$$

$$\mathbb{P}(S_T > 0 | S_\tau = d, \text{ drift on } [\tau, T]) = \mathbb{P}(Z > -d | Z \sim \mathcal{N}(-\mu[T-\tau], \sigma\sqrt{T-\tau}))$$

where $\mu = \dfrac{1}{2}u$.

Thus, the difference between those two probabilities is

$$\Phi\left(\frac{d}{\sigma\sqrt{[T-\tau]}}\right)-\Phi\left(\frac{d-[T-\tau]u/2}{\sigma\sqrt{[T-\tau]}}\right)$$

Thus, the optimal time τ is solution of

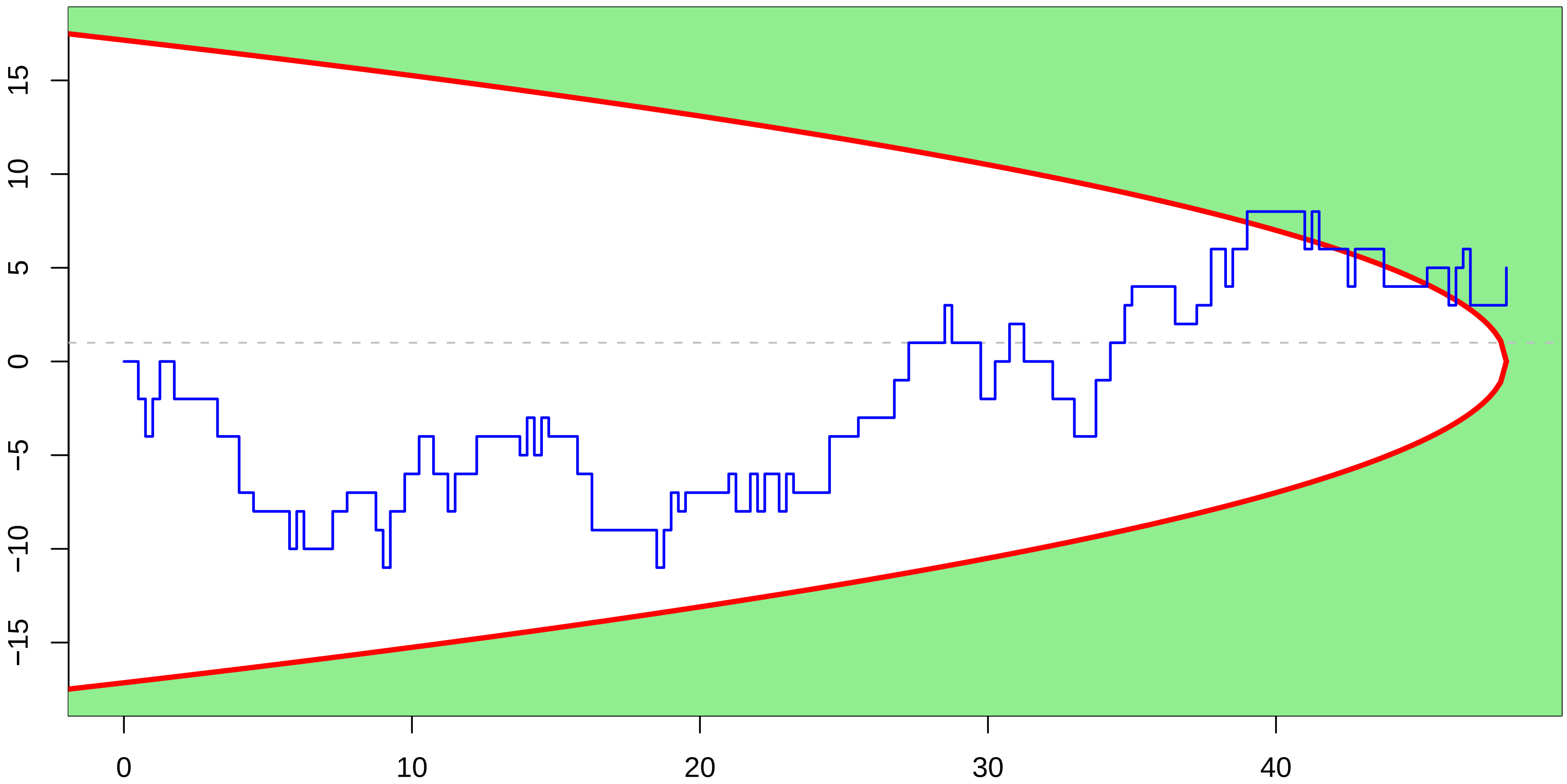
$$[L(\alpha-u)-L(\alpha)]\underbrace{\frac{[e^{-\delta\tau}-e^{-\delta T}]}{\delta}}_{\approx T-\tau}=\Phi\left(\frac{d}{\sigma\sqrt{[T-\tau]}}\right)-\Phi\left(\frac{d+[T-\tau]\gamma}{\sigma\sqrt{[T-\tau]}}\right).$$

i.e.

$$\tau=h(d,\lambda,u,L,\sigma).$$

Thus, the optimal time to stop playing (as a function of the remaining time $T-\tau$ and the score difference d) is the following region,

Region where teams stop making efforts



Obviously, it is too simple.... we need to consider a non-cooperative **game**.

Optimal strategy on a discretized version of the game

Assume that controls u_1 and u_2 are discrete, taking values in a set $\mathcal{U}$. Since we consider a non-null sum game, Nash equilibrium have to be searched in extremal points of polytopes of payoff matrices (see).

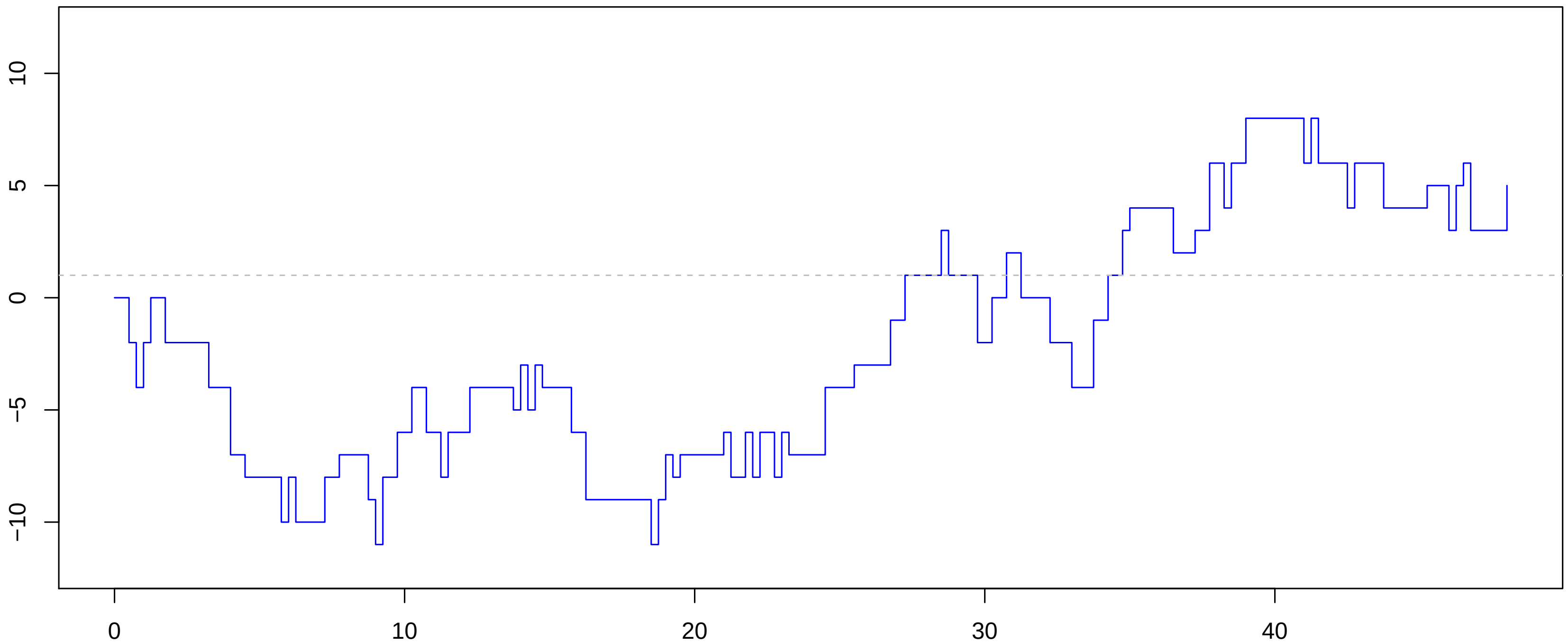
Looking for Nash equilibriums might not be a great strategy

Here, $(u_1^\star, u_2^\star)$ is solution of maxmin problems

$$u_1^\star \in \underset{u_1\in\mathcal{U}}{\text{argmax}}\left\{\min_{u_2\in\mathcal{U}} J_1(u_1,u_2)\right\} \text{ and } u_2^\star \in \underset{u_2\in\mathcal{U}}{\text{argmax}}\left\{\min_{u_1\in\mathcal{U}} J_2(u_1,u_2)\right\}$$

where J functions are payoffs.

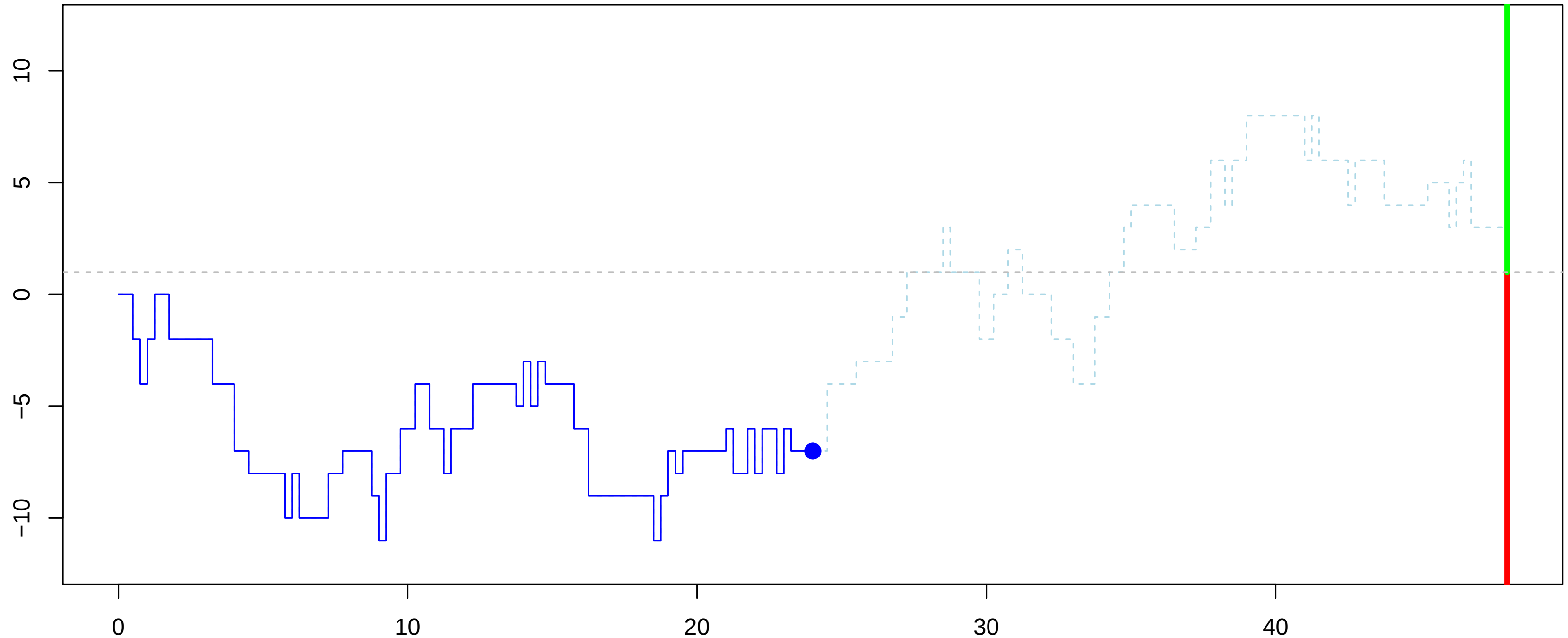
Discretized version of the stochastic game



Let $(S_t)_{t\in[0,T]}$ denote the score difference over the game,

$$dS^\star_t = (u^\star_1(S^\star_t) - u^\star_2(S^\star_t))dt + dW_t$$

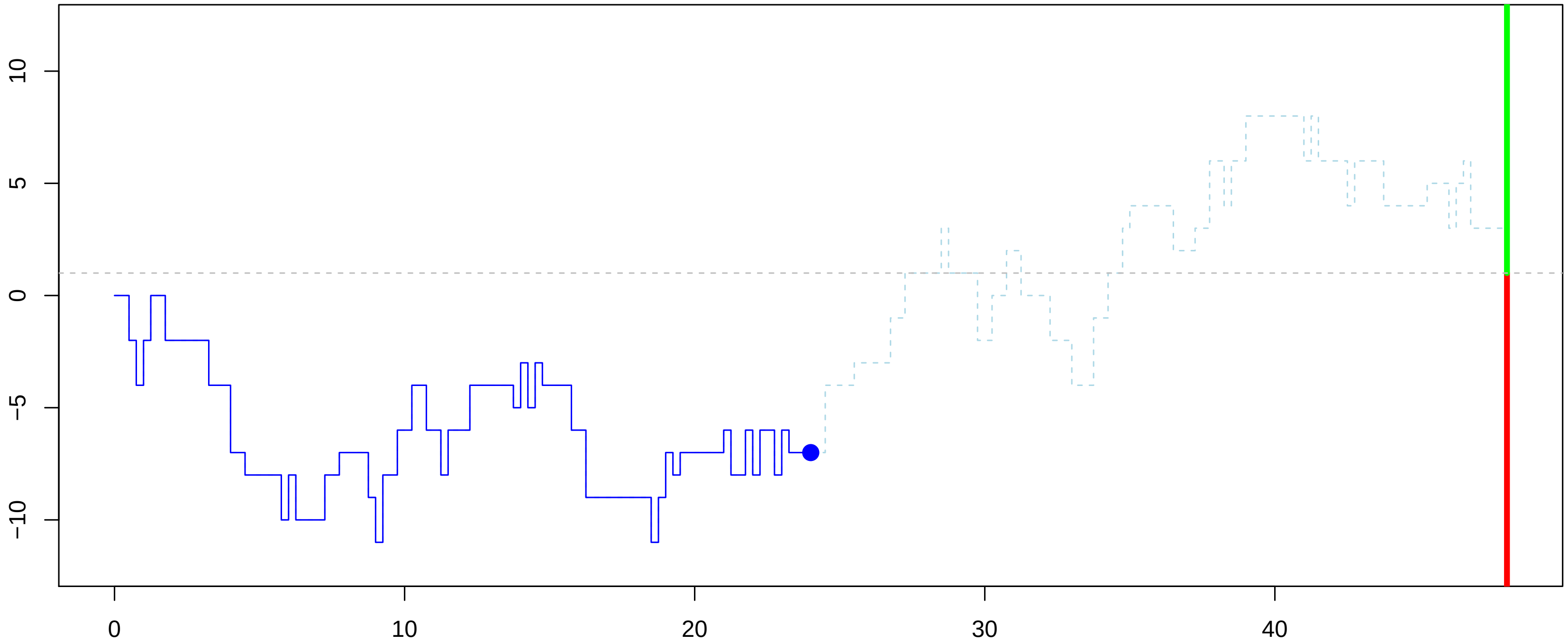
Discretized version of the stochastic game



At time $\tau \in [0,T)$, given $S_\tau = x$, player 1 seeks an optimal strategy,

$$u^\star_{1,\tau}(x) \in \underset{u_1\in\mathcal{U}}{\text{argmax}}\left\{\min_{u_2\in\mathcal{U}} \mathbb{E}\left(\alpha_1\mathbf{1}(S^\star_T>0)+\int_\tau^T L_1(u^\star_{1,s}(S^\star_s))ds\right)\right\}$$

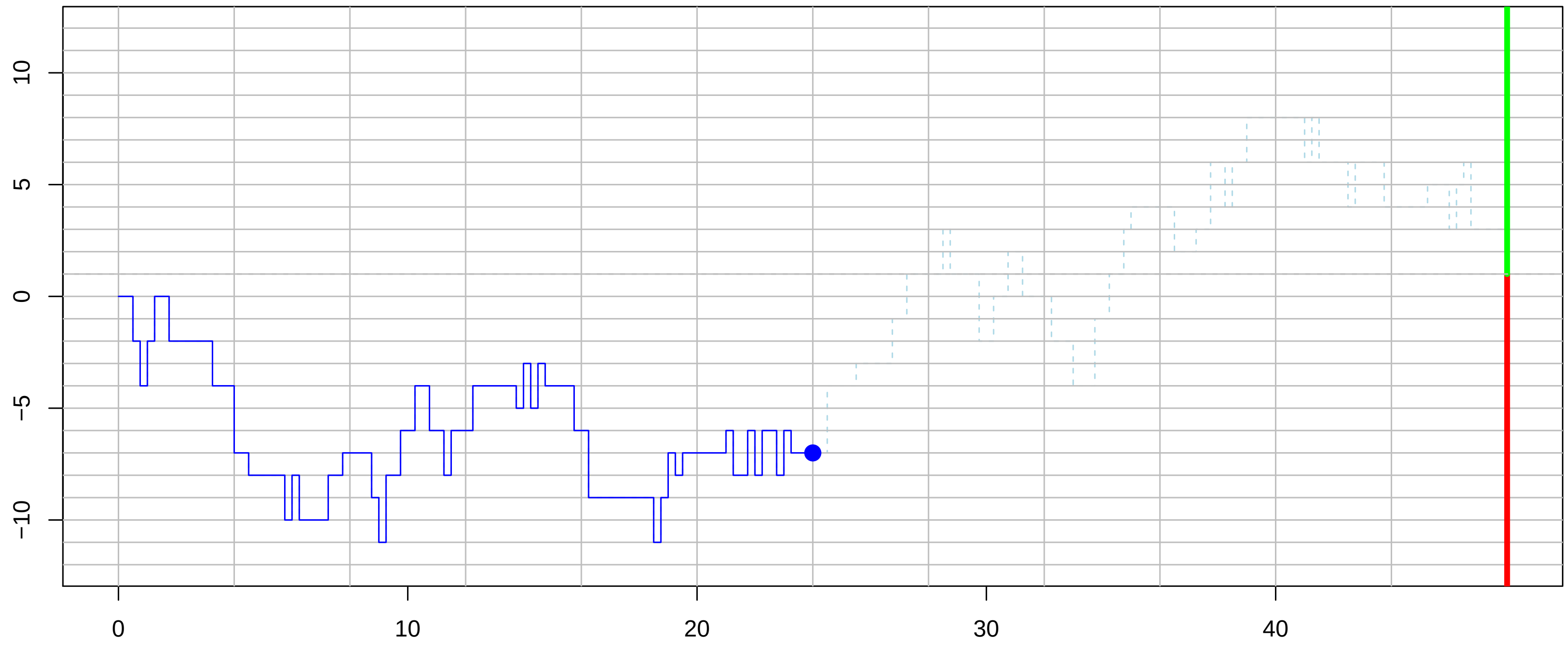
Discretized version of the stochastic game



At time $\tau \in [0,T)$, given $S_\tau = x$, player 1 seeks an optimal strategy,

$$u^\star_{1,\tau}(x) \in \underset{u_1 \in \mathcal{U}}{\text{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} \mathbb{E} \left(\alpha_1 \mathbf{1}(S^\star_T > 0) + \int_\tau^{\tau+h} L_1(u_1) ds + \int_{\tau+h}^T L_1(u^\star_{1,s}(S^\star_s)) ds \right) \right\}$$

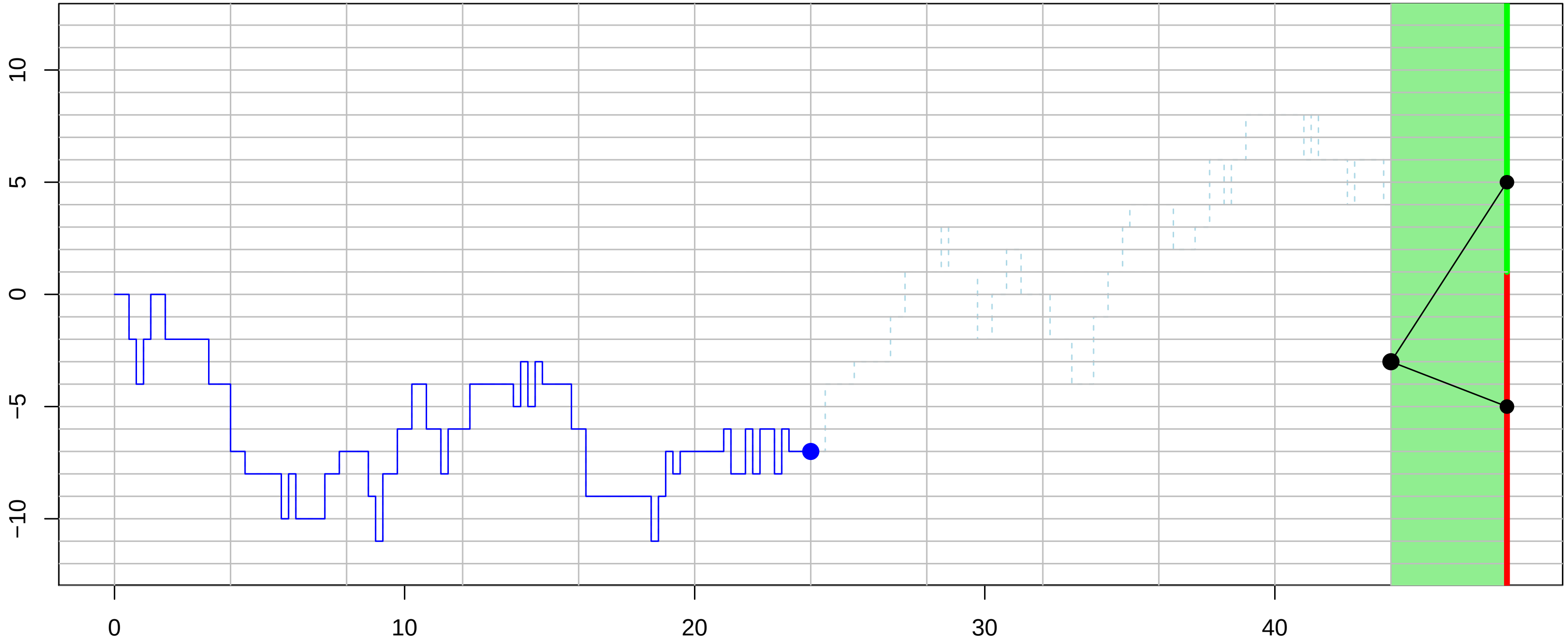
Discretized version of the stochastic game



Consider a discretization of $[0, T]$ so that optimal controls can be updated at times t_k where $0 = t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_{n-2} \leq t_{n-1} \leq t_n = T$.

We solve the problem backward, starting at time t_{n-1}.

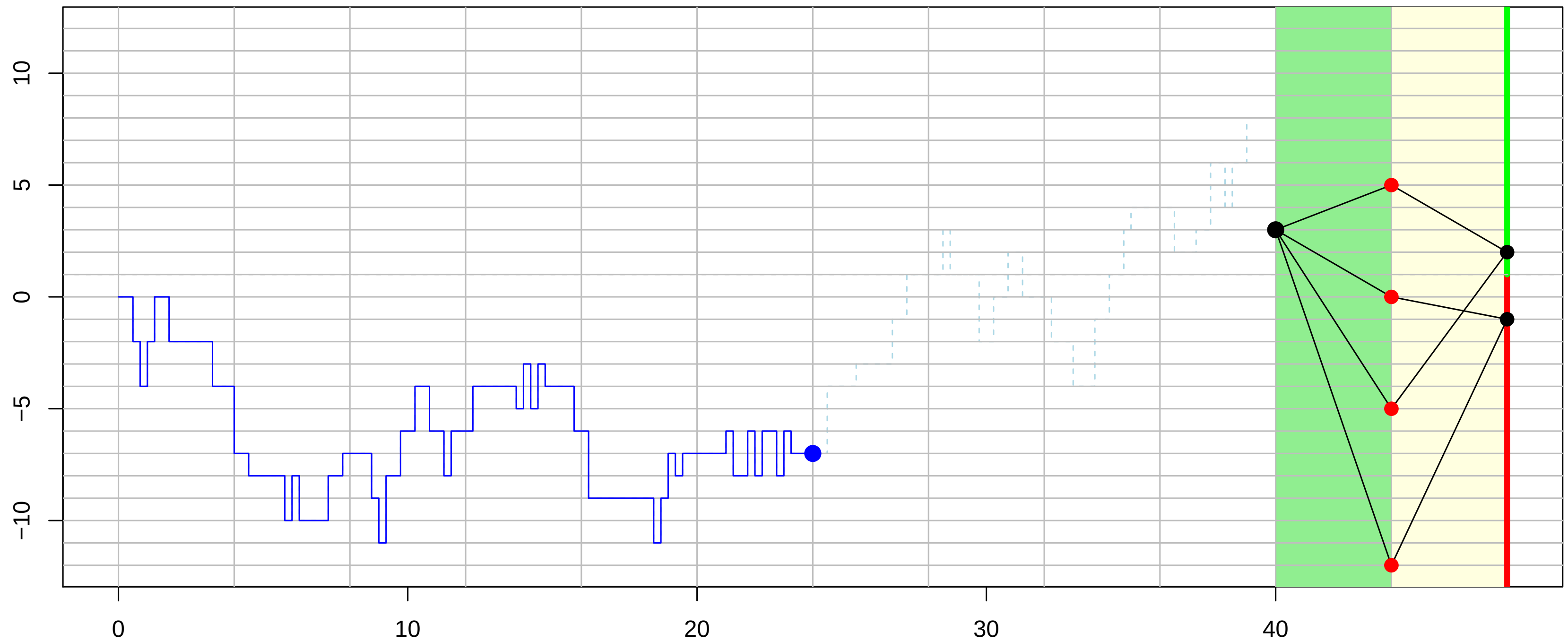
Discretized version of the stochastic game



Given controls (u_1, u_2) , $S_{t_n} = S_{t_{n-1}} + [u_1 - u_2](t_n - t_{n-1}) + \varepsilon_n$, where $S_{t_{n-1}} = x$.

$u^\star_{1,n-1}(x) \in \underset{u_1 \in \mathcal{U}}{\text{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} J_1(u_1, u_2) \right\}$ where $J_1(u_1, u_2)$ is the sum of two terms, $\mathbb{P}(S_{t_n} > 0 | S_{t_{n-1}} = x) = \sum_{s \in \mathcal{S}_+} \mathbb{P}(S_{t_n} = s | S_{t_{n-1}} = x)$ and $L_1(u_1)$.

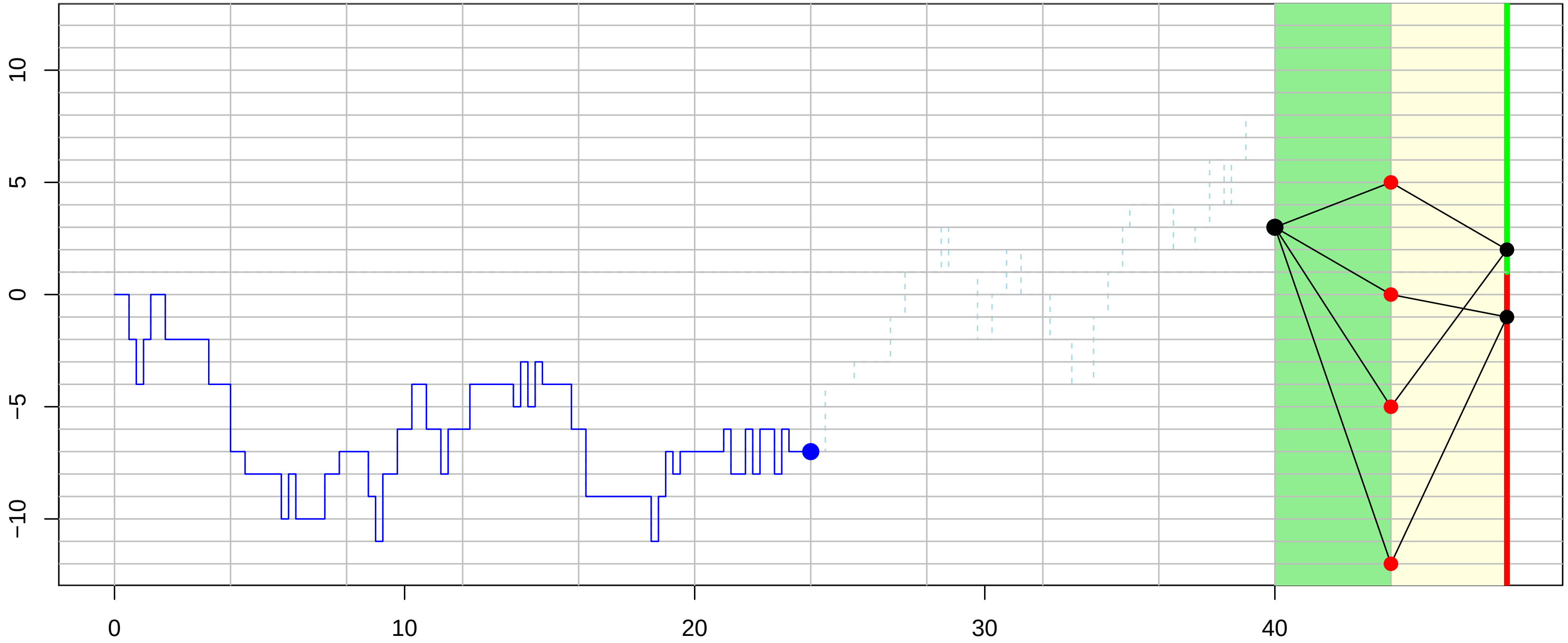
Discretized version of the stochastic game



$$S_{t_n} = \underbrace{S_{t_{n-2}} + [u_1 - u_2](t_{n-1} - t_{n-2}) + \varepsilon_{n-1}}_{S_{t_{n-1}}} + [u^\star_{1,n-1} - u^\star_{2,n-1}(S_{t_{n-1}})](t_n - t_{n-1}) + \varepsilon_n,$$

where $S_{t_{n-2}} = x$. Here $u^\star_{1,n-2}(x) \in \underset{u_1 \in \mathcal{U}}{\text{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} J_1(u_1, u_2) \right\}$, where $J_1(u_1, u_2)$...

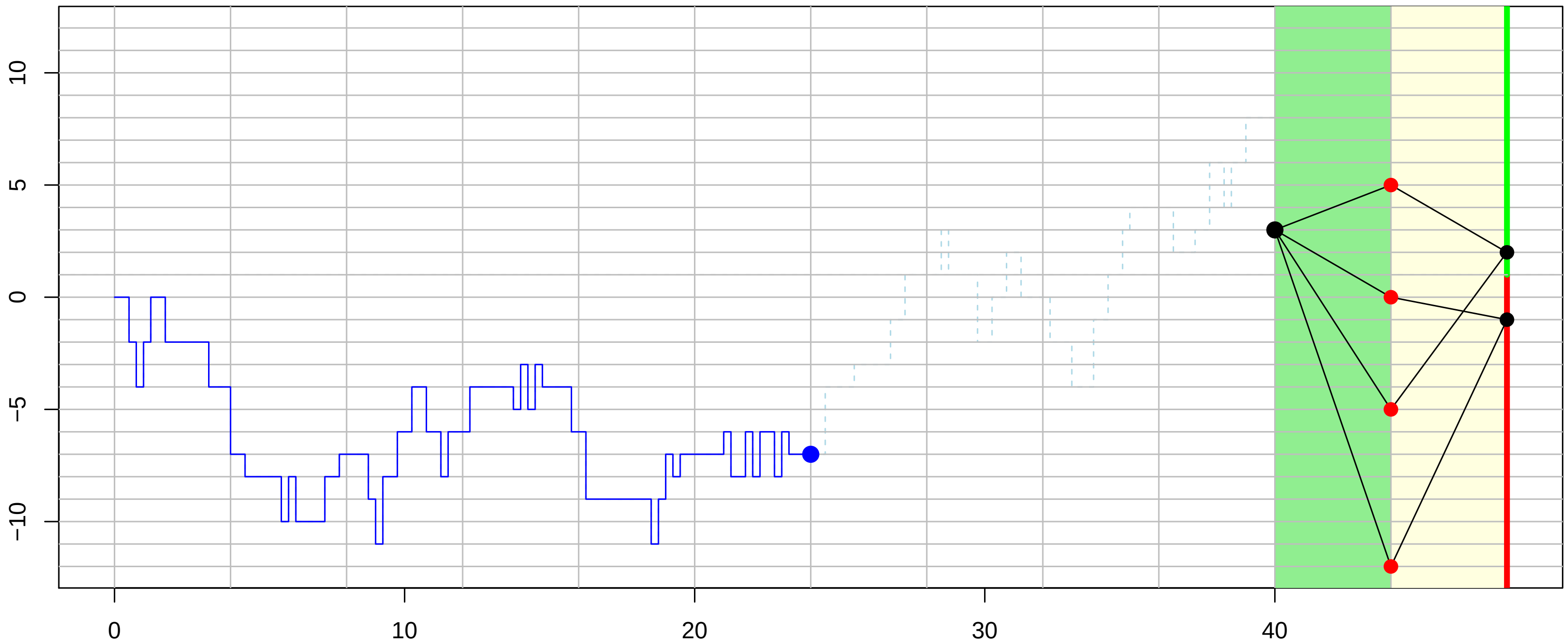
Discretized version of the stochastic game



... is the sum of two terms, based on

$$\mathbb{P}(S_{t_n}=y|S_{t_{n-2}}=x)=\sum_{s\in\mathcal{S}}\underbrace{\mathbb{P}(S_{t_n}=y|S_{t_{n-1}}=s)}_{\text{function of }(u^\star_{1,n-1}(s),u^\star_{2,n-1}(s))}\cdot\underbrace{\mathbb{P}(S_{t_{n-1}}=s|S_{t_{n-2}}=x)}_{\text{function of }(u_1,u_2)}$$

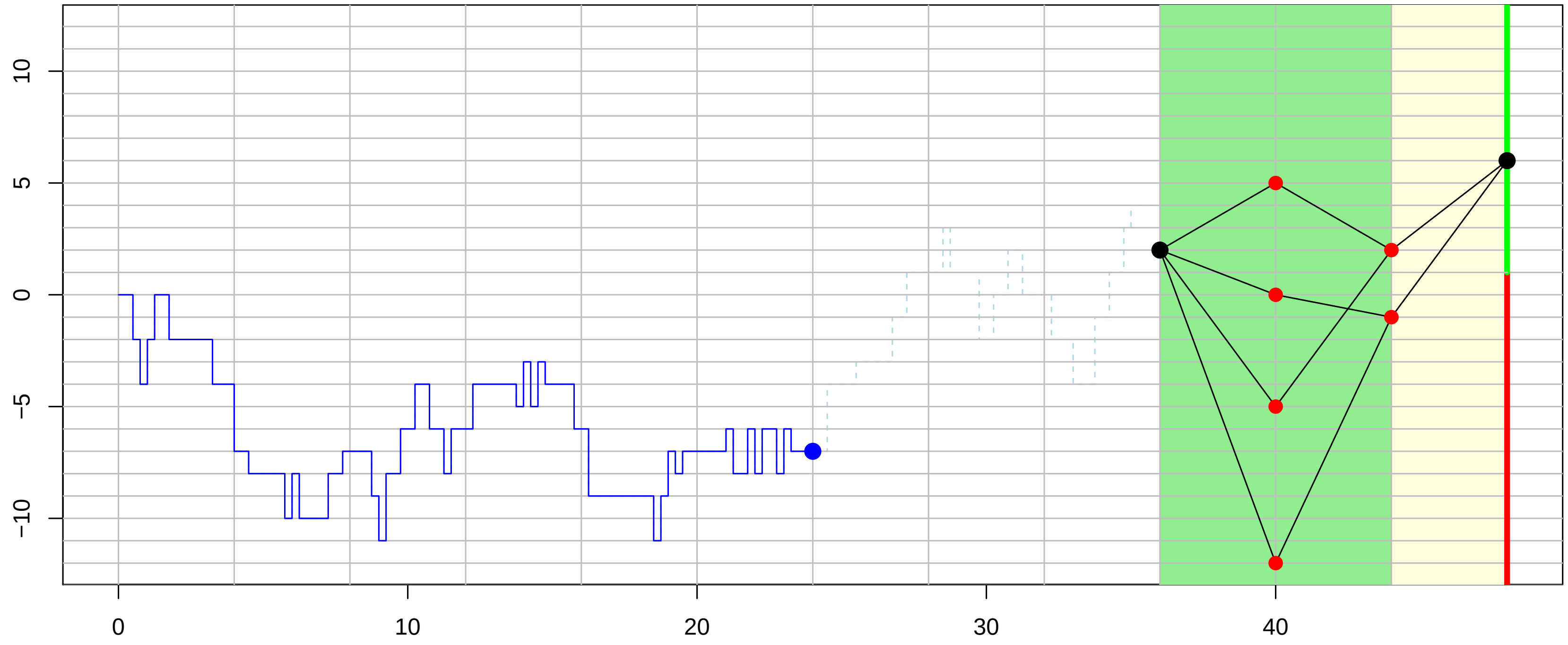
Discretized version of the stochastic game



... one term is $\mathbb{P}(S_{t_n} > 0 | S_{t_{n-2}} = x)$ (as before), the sum of $L_1(u_1)$ and

$$\mathbb{E}(L_1(u^\star_{1,n-1})) = \sum_{s\in\mathcal{S}} L_1(u^\star_{1,n-1}(s)) \cdot P(S_{t_{n-1}} = s | S_{t_{n-2}} = x)$$

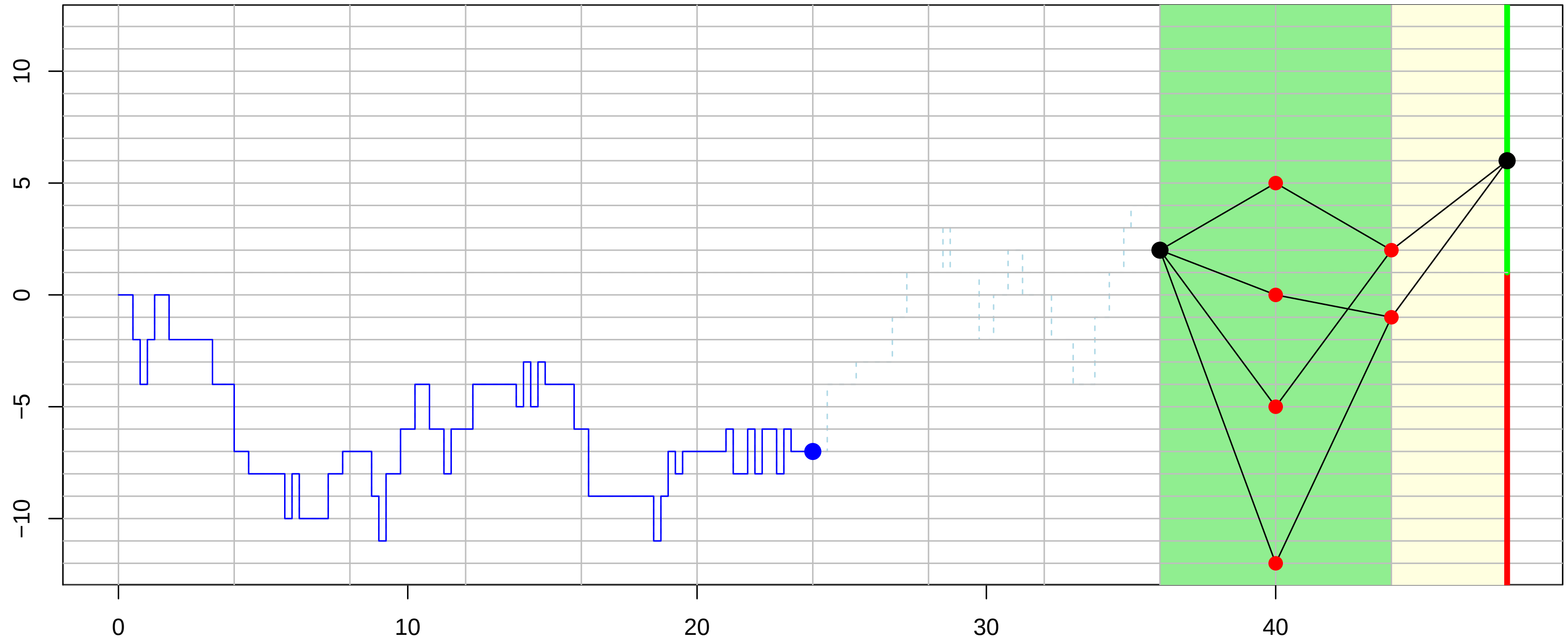
Discretized version of the stochastic game



$$S_{t_n} = \underbrace{\underbrace{S_{t_{n-3}} + [u_1 - u_2]dt + \varepsilon_{n-2}}_{S_{t_{n-2}}} + [u^\star_{1,n-2} - u^\star_{2,n-2}(S_{t_{n-2}})]dt + \varepsilon_{n-1}}_{S_{t_{n-1}}}$$

$$+[u^\star_{1,n-1} - u^\star_{2,n-1}(S_{t_{n-1}})]dt + \varepsilon_n \text{ with } S_{t_{n-3}} = x.$$

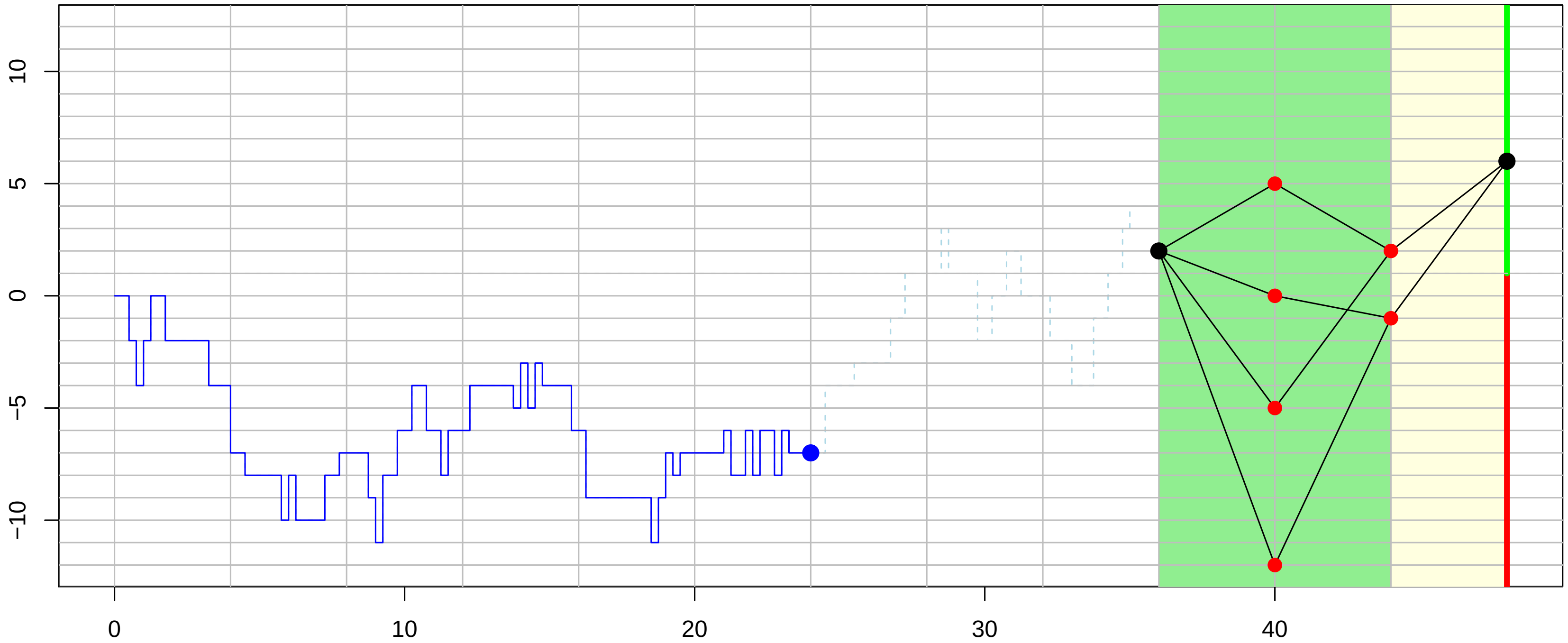
Discretized version of the stochastic game



$$\mathbb{P}(S_{t_n}=y|S_{t_{n-3}}=x)=\sum_{s_1,s_2\in\mathcal{S}}\underbrace{\mathbb{P}(S_{t_n}=y|S_{t_{n-1}}=s_2)}_{\text{function of }(u^\star_{1,n-1}(s_2),u^\star_{2,n-1}(s_2))}$$

$$\cdot\underbrace{\mathbb{P}(S_{t_{n-1}}=s_2|S_{t_{n-2}}=s_1)}_{\text{function of }(u^\star_{1,n-2}(s_1),u^\star_{2,n-2}(s_1))}\cdot\underbrace{\mathbb{P}(S_{t_{n-2}}=s_2|S_{t_{n-3}}=s_1)}_{\text{function of }(u_1,u_2)}$$

Discretized version of the stochastic game



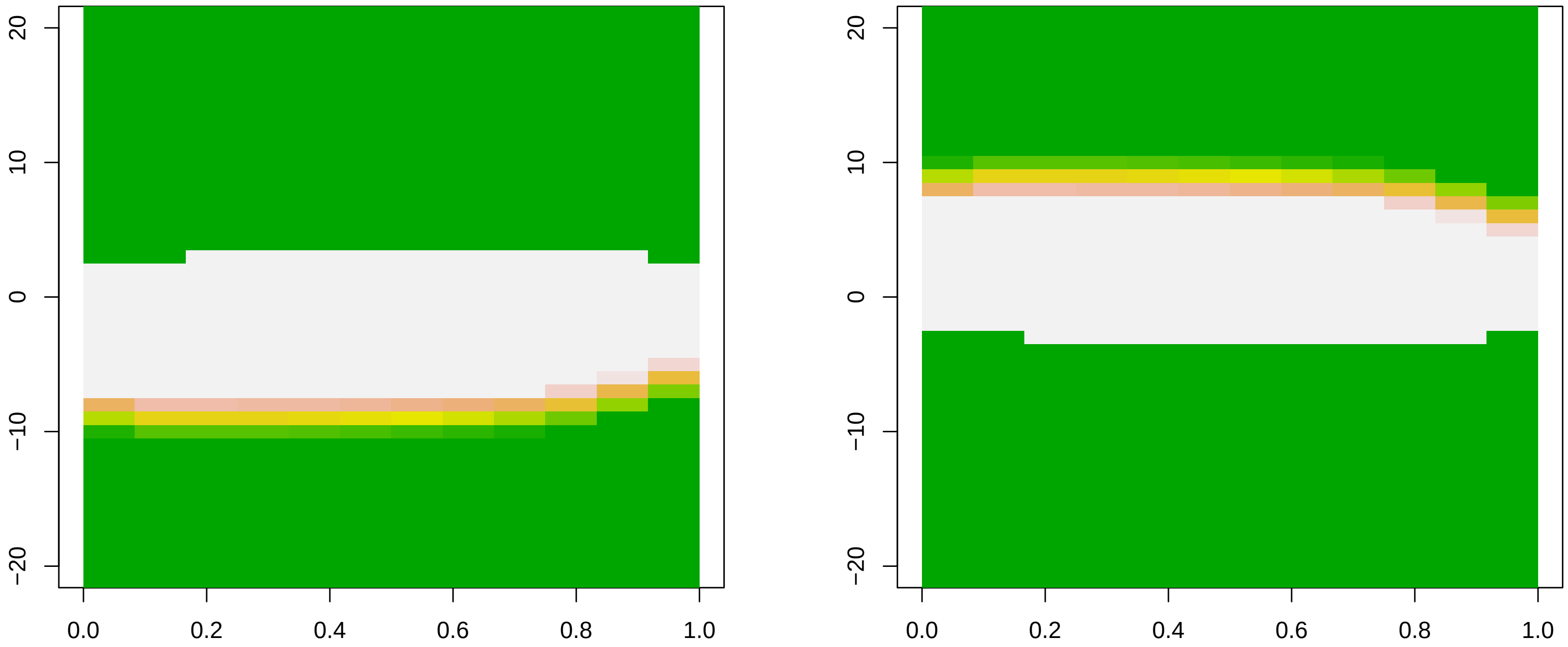
Based on those probabilities, we have $\mathbb{P}(S_{t_n} > 0 | S_{t_{n-3}} = x)$ and the second term is the sum of $L_1(u_1)$ and $\mathbb{E}(L_1(u^\star_{1,n-2}) + L_1(u^\star_{1,n-1}))$ i.e.

$$\sum_{s\in\mathcal{S}} L_1(u^\star_{1,n-2}(s)) \cdot P(S_{t_{n-2}} = s | S_{t_{n-3}} = x) + \sum_{s\in\mathcal{S}} L_1(u^\star_{1,n-1}(s)) \cdot P(S_{t_{n-1}} = s | S_{t_{n-3}} = x)$$

Numerical computation of the discretized game

team 1 on the left vs team 2 on the right : ■ low effort □ high effort

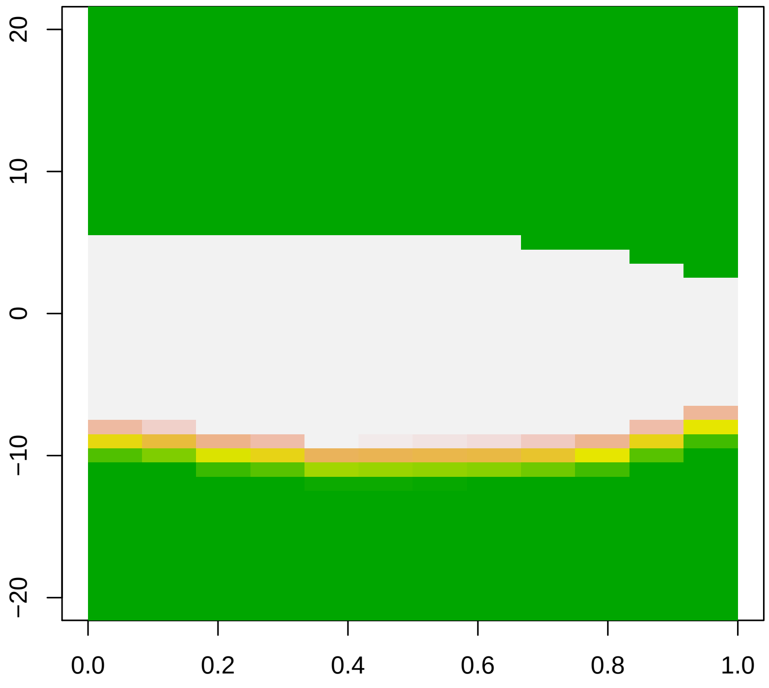
(simple numerical application, with $\#\mathcal{U} = 60$ and $n = 12$)

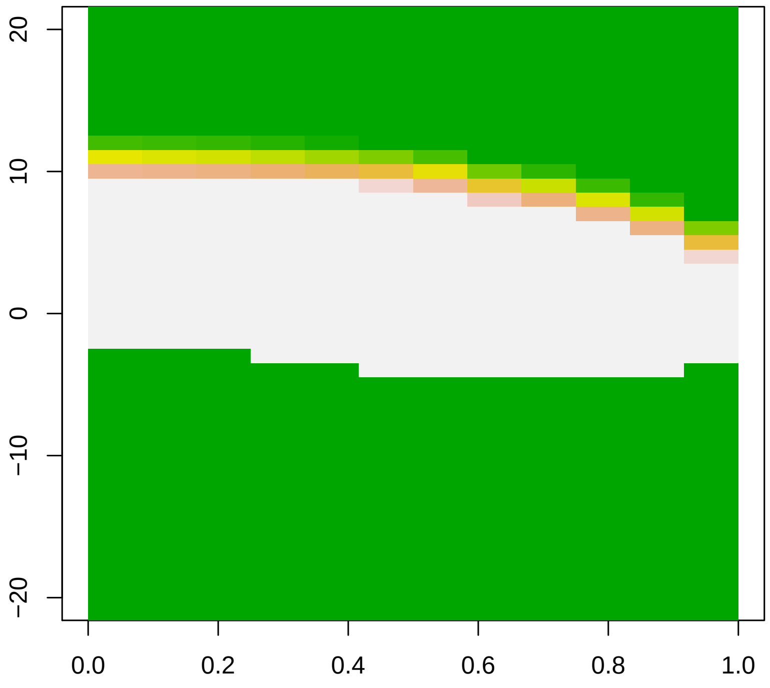


Numerical computation of the discretized game

team 1 on the left vs team 2 on the right : ■ low effort □ high effort $\alpha_1 \uparrow$

$$u^\star_{1,\tau}(x) \in \underset{u_1 \in \mathcal{U}}{\text{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} \mathbb{E}\left(\alpha_1 \mathbf{1}(S^\star_T > 0) + \int_\tau^T e^{-\delta_1(s-\tau)}(b_1 - u^\star_{1,s}(S^\star_s))^{\gamma_1} ds \right) \right\}$$

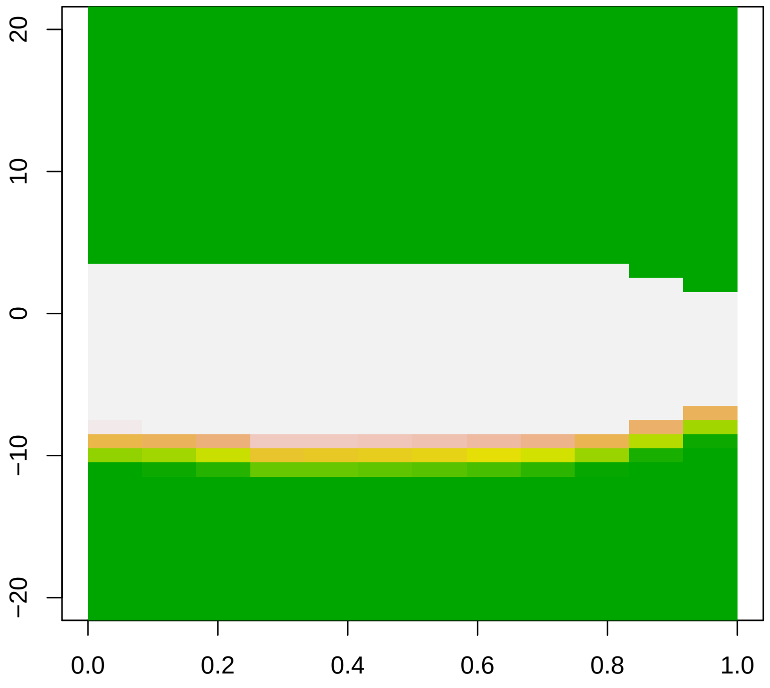


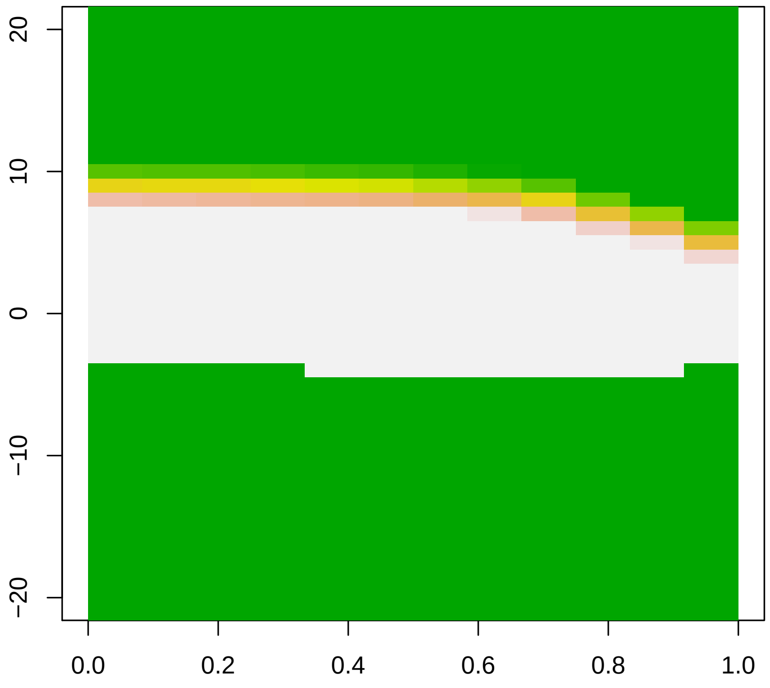


Numerical computation of the discretized game

team 1 on the left vs team 2 on the right : ■ low effort □ high effort $b_1 \uparrow$

$$u^\star_{1,\tau}(x) \in \operatorname*{argmax}_{u_1 \in \mathcal{U}} \left\{ \min_{u_2 \in \mathcal{U}} \mathbb{E} \left(\alpha_1 \mathbf{1}(S^\star_T > 0) + \int_\tau^T e^{-\delta_1(s-\tau)} (b_1 - u^\star_{1,s}(S^\star_s))^{\gamma_1} ds \right) \right\}$$

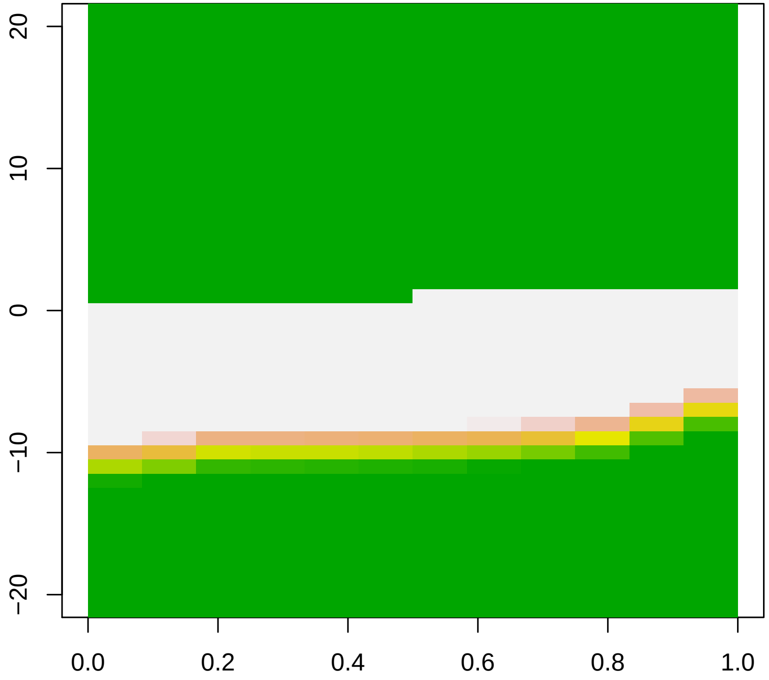


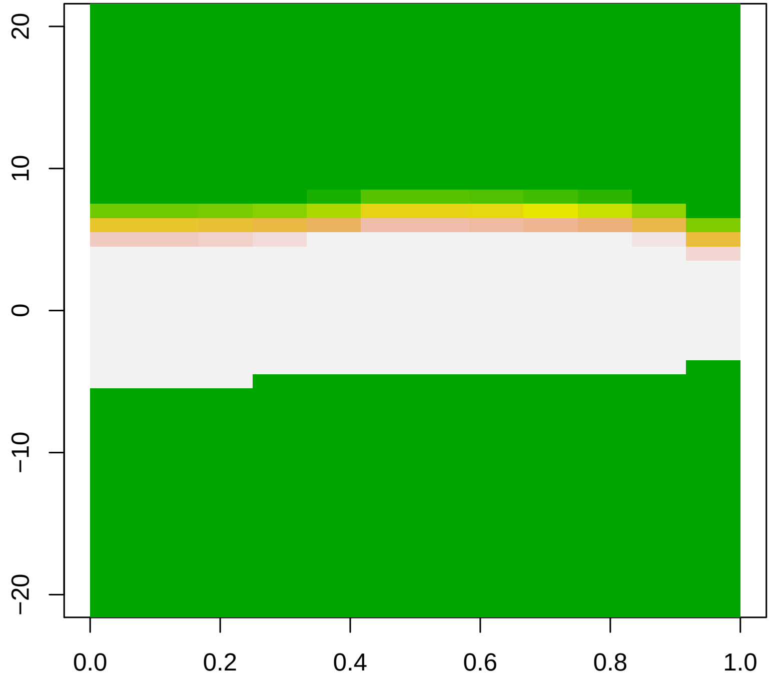


Numerical computation of the discretized game

team 1 on the left vs team 2 on the right : ■ low effort □ high effort $\gamma_1 \uparrow$

$$u_{1,\tau}^{\star}(x) \in \underset{u_1 \in \mathcal{U}}{\operatorname{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} \mathbb{E}\left(\alpha_1 \mathbf{1}(S_T^{\star} > 0) + \int_{\tau}^{T} e^{-\delta_1 (s-\tau)} (b_1 - u_{1,s}^{\star}(S_s^{\star}))^{\gamma_1} ds \right) \right\}$$

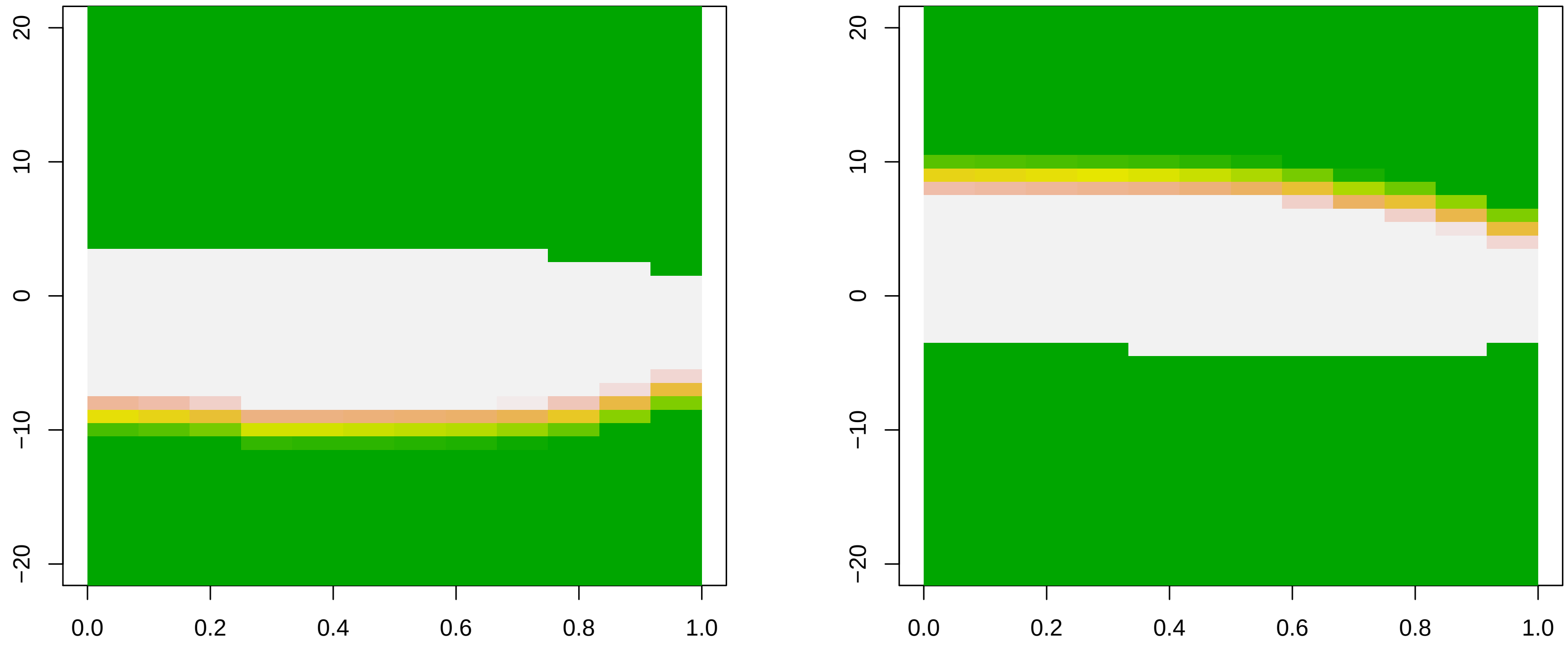




Numerical computation of the discretized game

team 1 on the left vs team 2 on the right : ■ low effort □ high effort $\delta_1 \uparrow$

$$u^\star_{1,\tau}(x) \in \underset{u_1 \in \mathcal{U}}{\text{argmax}} \left\{ \min_{u_2 \in \mathcal{U}} \mathbb{E}\left(\alpha_1 \mathbf{1}(S^\star_T > 0) + \int_\tau^T e^{-\delta_1(s-\tau)}(b_1 - u^\star_{1,s}(S^\star_s))^{\gamma_1} ds \right) \right\}$$



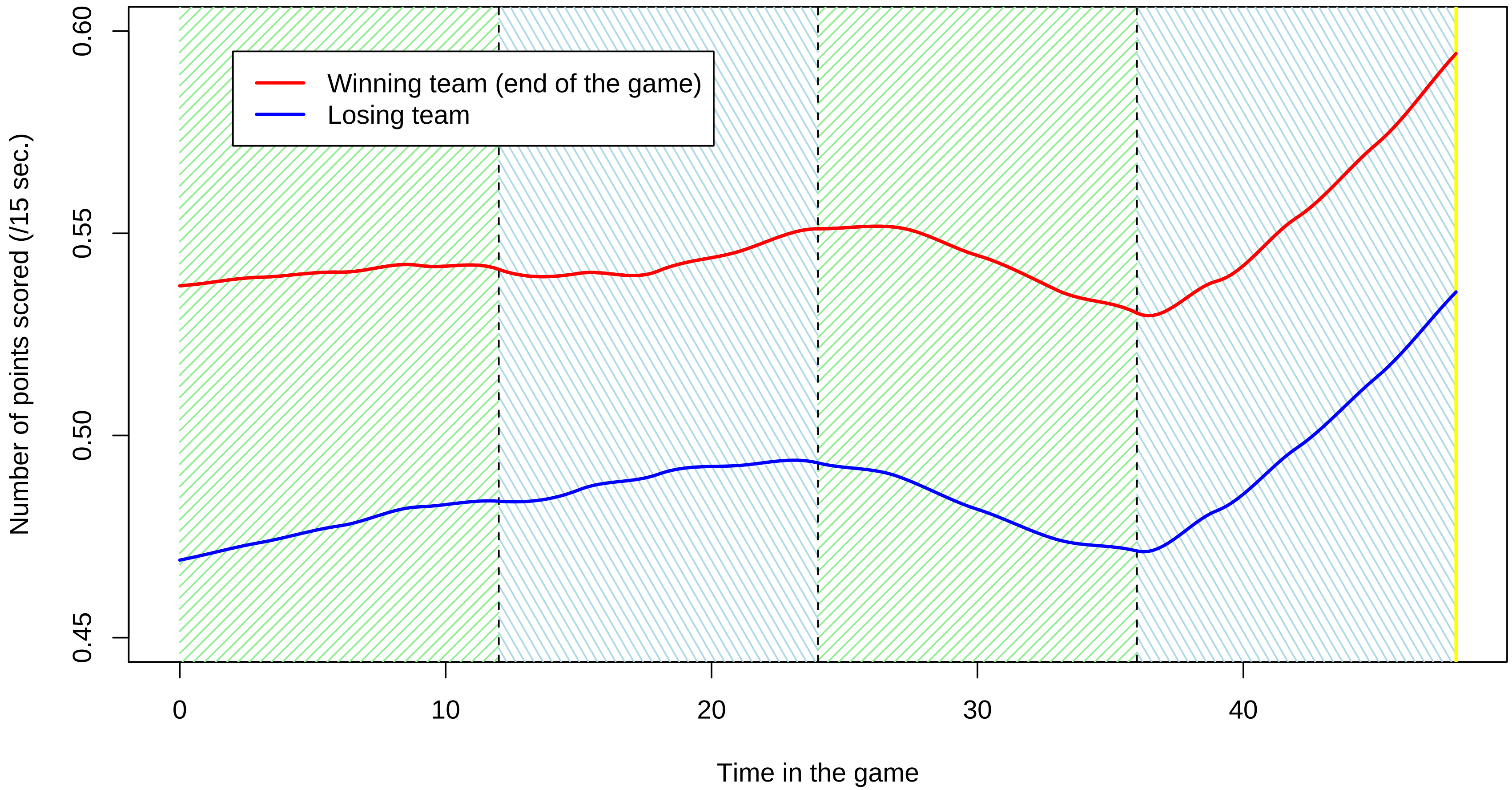
Description of the data

GameID	LineNumber	TimeRemaining	Entry
20081028CLEBOS	1	00:48:00	Start of 1st Quarter
20081028CLEBOS	2	00:48:00	Jump Ball Perkins vs Ilgauskas
20081028CLEBOS	3	00:47:40	[BOS] Rondo Foul:Shooting (1 PF)
20081028CLEBOS	4	00:47:40	[CLE 1-0] West Free Throw 1 of 2 (1 PTS)
20081028CLEBOS	5	00:47:40	[CLE 2-0] West Free Throw 2 of 2 (2 PTS)
20081028CLEBOS	6	00:47:30	[BOS] Garnett Jump Shot: Missed
20081028CLEBOS	7	00:47:28	[CLE] James Rebound (Off:0 Def:1)
20081028CLEBOS	8	00:47:22	[CLE 4-0] James Pullup Jump shot: Made (2 PTS)
20081028CLEBOS	9	00:47:06	[BOS 2-4] Pierce Slam Dunk Shot: Made (2 PTS) Assist: Rondo (1 AST)
20081028CLEBOS	10	00:46:57	[CLE] James 3pt Shot: Missed
20081028CLEBOS	11	00:46:56	[BOS] R. Allen Rebound (Off:0 Def:1)
20081028CLEBOS	12	00:46:47	[BOS 4-4] Garnett Slam Dunk Shot: Made (2 PTS) Assist: Rondo (2 AST)
20081028CLEBOS	13	00:46:24	[CLE 6-4] Ilgauskas Driving Layup Shot: Made (2 PTS) Assist: James (1 AST)
20081028CLEBOS	14	00:46:13	[BOS] Garnett Jump Shot: Missed
20081028CLEBOS	15	00:46:11	[BOS] Perkins Rebound (Off:1 Def:0)
20081028CLEBOS	16	00:46:08	[BOS] Pierce 3pt Shot: Missed
20081028CLEBOS	17	00:46:06	[CLE] Ilgauskas Rebound (Off:0 Def:1)
20081028CLEBOS	18	00:45:52	[CLE] M. Williams Layup Shot: Missed
20081028CLEBOS	19	00:45:51	[BOS] Garnett Rebound (Off:0 Def:1)
20081028CLEBOS	20	00:45:46	[BOS] R. Allen Layup Shot: Missed Block: James (1 BLK)
20081028CLEBOS	21	00:45:44	[CLE] West Rebound (Off:0 Def:1)

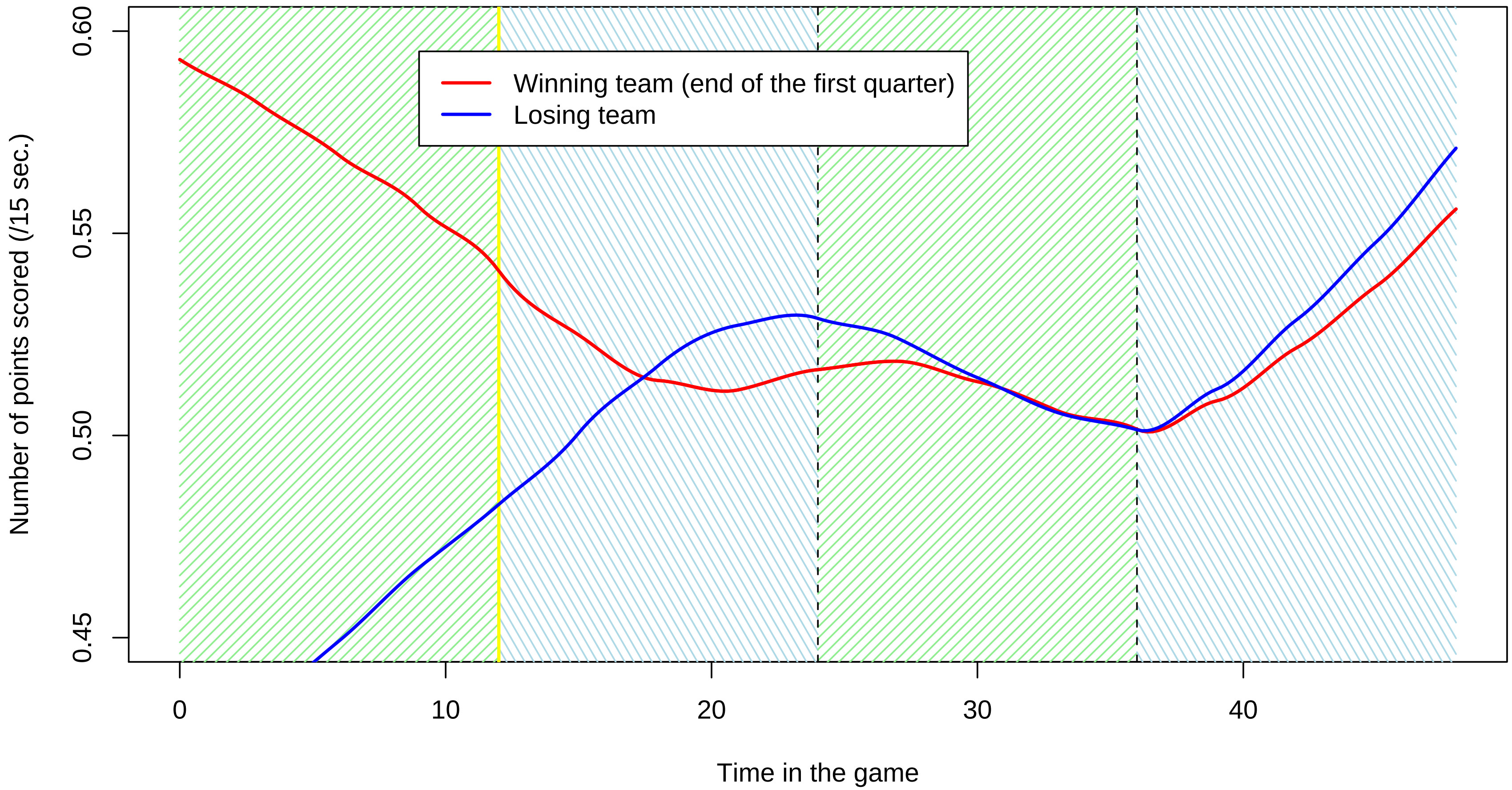
Description of the data

GameID	LineNumber	TimeRemaining	Entry
20081028CLEBOS	1	00:48:00	Start of 1st Quarter
20081028CLEBOS	2	00:48:00	Jump Ball Perkins vs Ilgauskas
20081028CLEBOS	3	00:47:40	[BOS] Rondo Foul:Shooting (1 PF)
20081028CLEBOS	4	00:47:40	[CLE 1-0] West Free Throw 1 of 2 (1 PTS)
20081028CLEBOS	5	00:47:40	[CLE 2-0] West Free Throw 2 of 2 (2 PTS)
20081028CLEBOS	6	00:47:30	[BOS] Garnett Jump Shot: Missed
20081028CLEBOS	7	00:47:28	[CLE] James Rebound (Off:0 Def:1)
20081028CLEBOS	8	00:47:22	[CLE 4-0] James Pullup Jump shot: Made (2 PTS)
20081028CLEBOS	9	00:47:06	[BOS 2-4] Pierce Slam Dunk Shot: Made (2 PTS) Assist: Rondo (1 AST)
20081028CLEBOS	10	00:46:57	[CLE] James 3pt Shot: Missed
20081028CLEBOS	11	00:46:56	[BOS] R. Allen Rebound (Off:0 Def:1)
20081028CLEBOS	12	00:46:47	[BOS 4-4] Garnett Slam Dunk Shot: Made (2 PTS) Assist: Rondo (2 AST)
20081028CLEBOS	13	00:46:24	[CLE 6-4] Ilgauskas Driving Layup Shot: Made (2 PTS) Assist: James (1 AST)
20081028CLEBOS	14	00:46:13	[BOS] Garnett Jump Shot: Missed
20081028CLEBOS	15	00:46:11	[BOS] Perkins Rebound (Off:1 Def:0)
20081028CLEBOS	16	00:46:08	[BOS] Pierce 3pt Shot: Missed
20081028CLEBOS	17	00:46:06	[CLE] Ilgauskas Rebound (Off:0 Def:1)
20081028CLEBOS	18	00:45:52	[CLE] M. Williams Layup Shot: Missed
20081028CLEBOS	19	00:45:51	[BOS] Garnett Rebound (Off:0 Def:1)
20081028CLEBOS	20	00:45:46	[BOS] R. Allen Layup Shot: Missed Block: James (1 BLK)
20081028CLEBOS	21	00:45:44	[CLE] West Rebound (Off:0 Def:1)

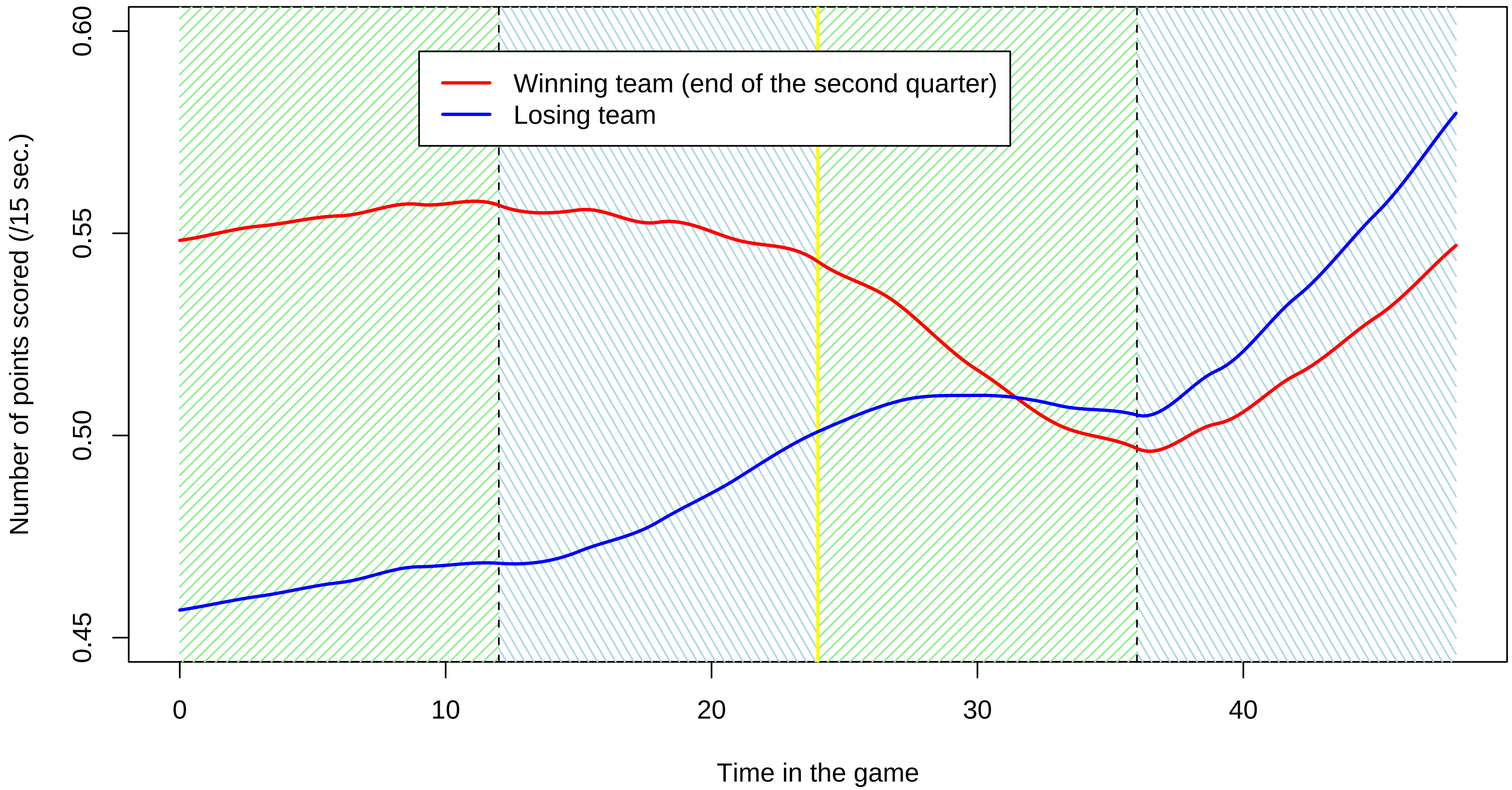
Homogeneity of the scoring process



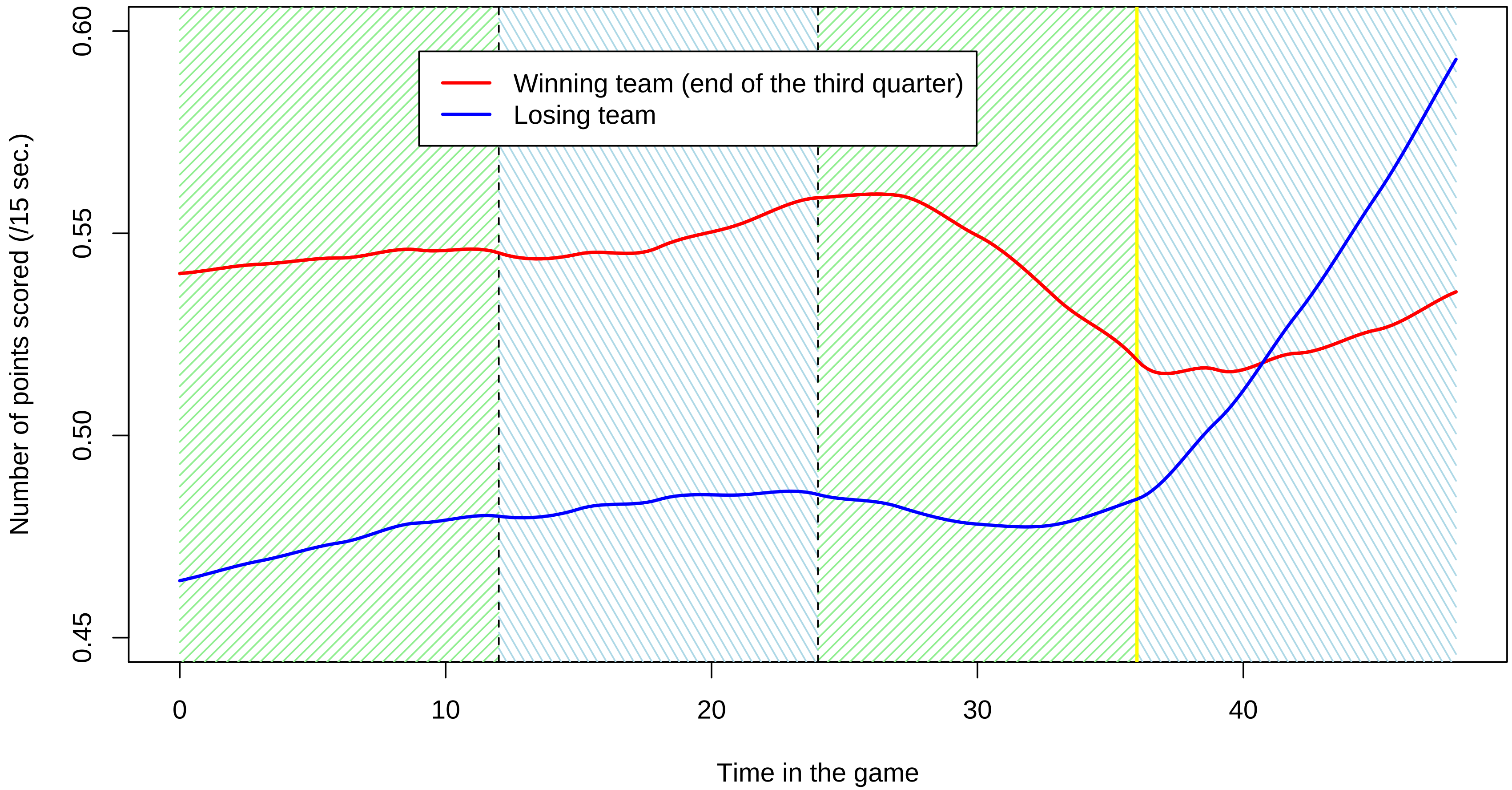
The scoring process : ex post analysis of the score



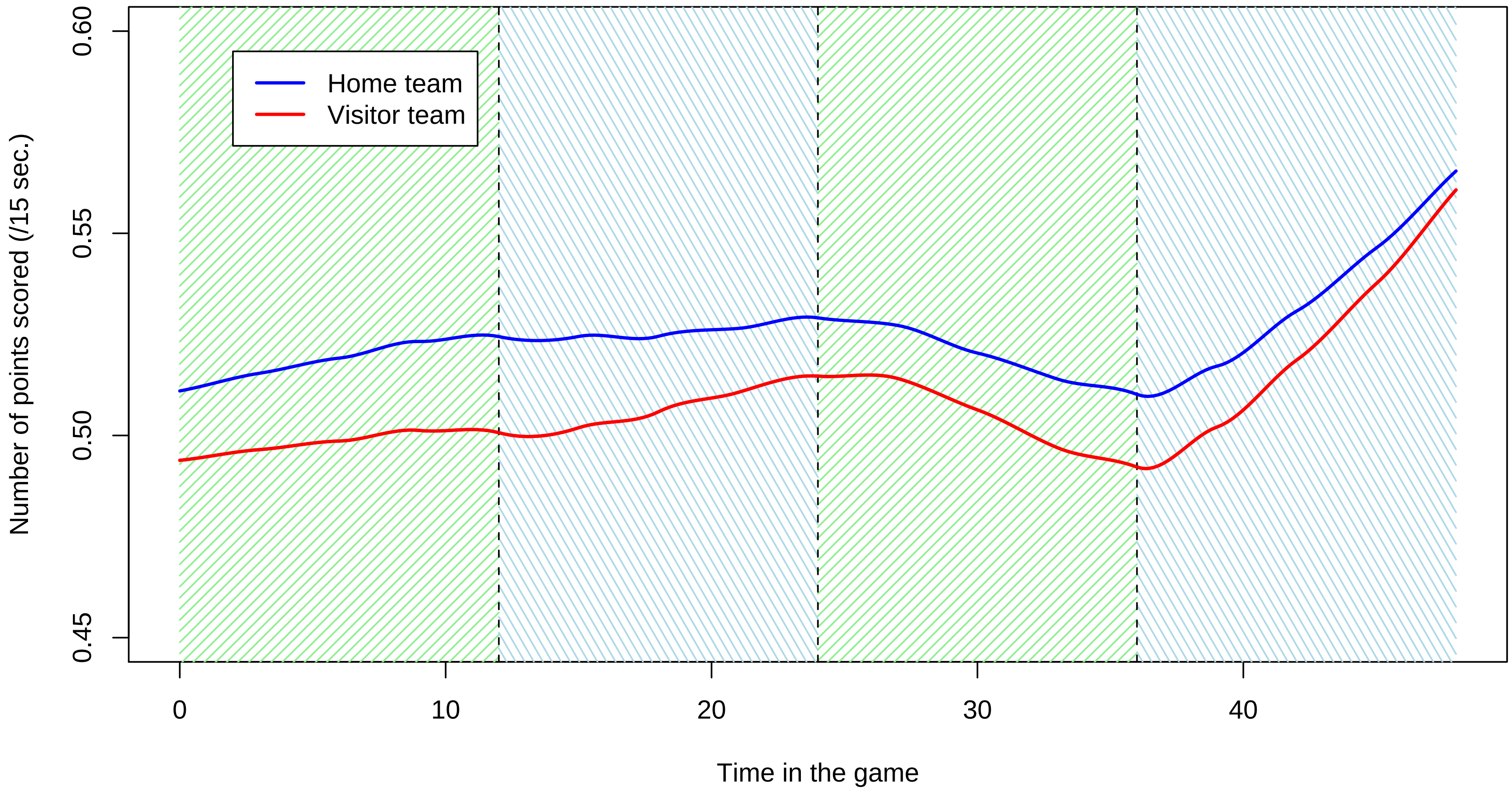
The scoring process : ex post analysis of the score



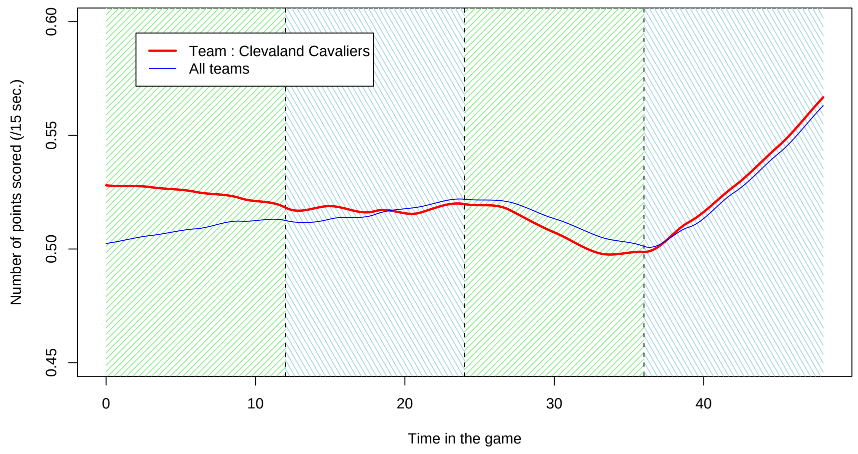
The scoring process : ex post analysis of the score

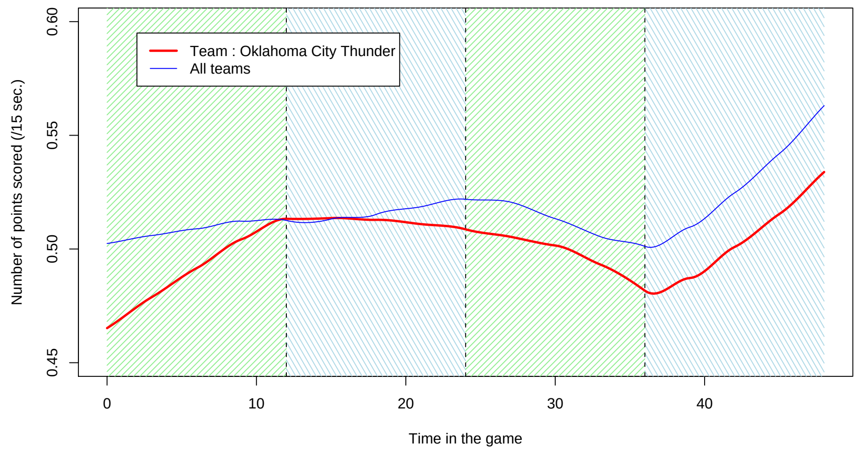


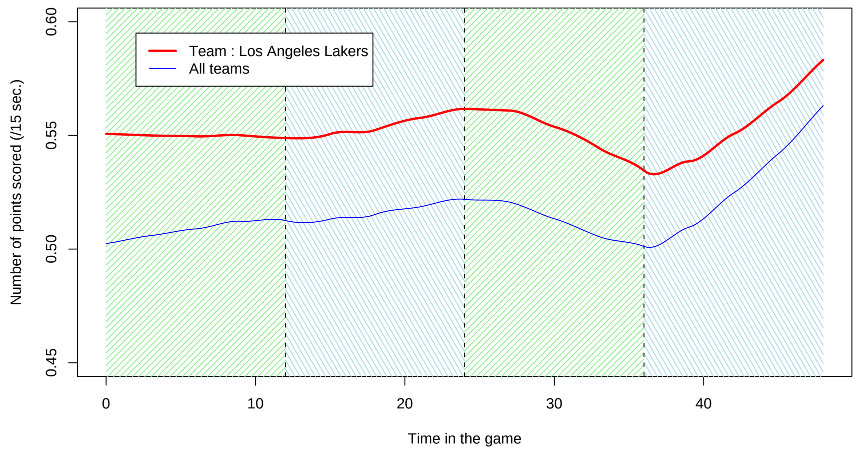
The scoring process : home versus visitor

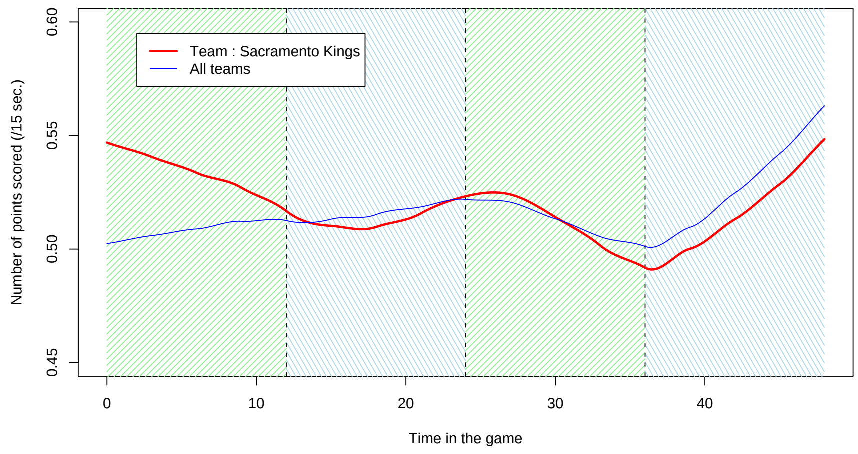


The scoring process : team strategies ?

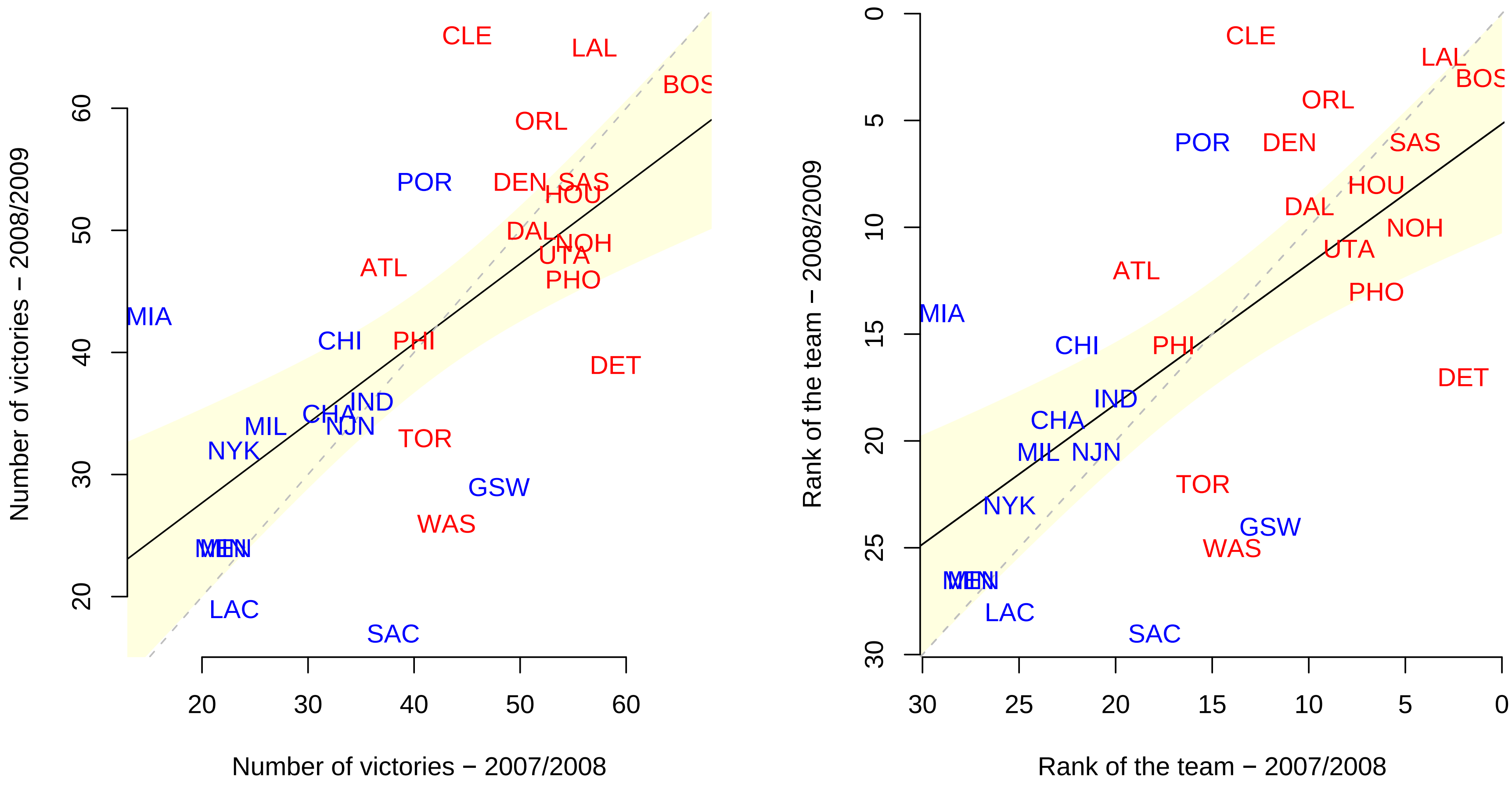








Effect of explanatory variables ?



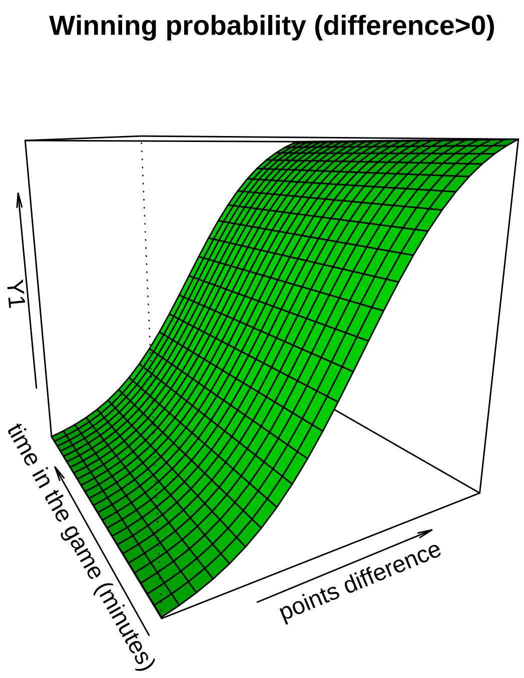
cf. Galton's *regression to the mean.*

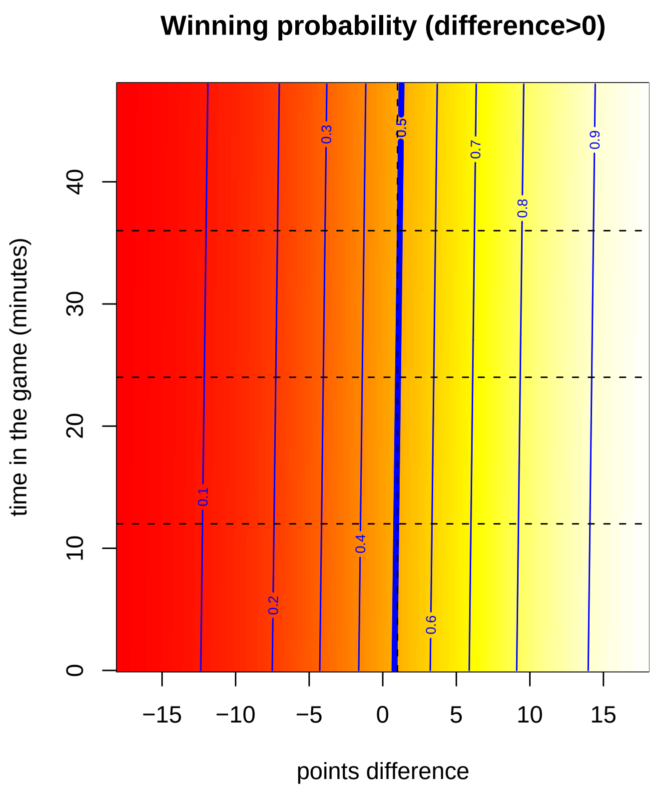
Winning as a function of time and score difference

Following the idea of Berger and Pope (2009),

$$\text{win}_i = \alpha + \beta(\text{score difference})_{i,t} + \gamma(\text{time in the game})_t + \delta X_i + \varepsilon_i$$

(simple linear model)



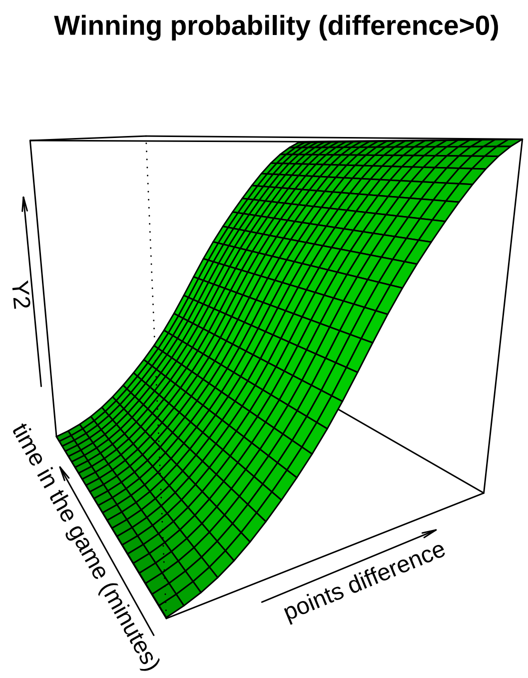


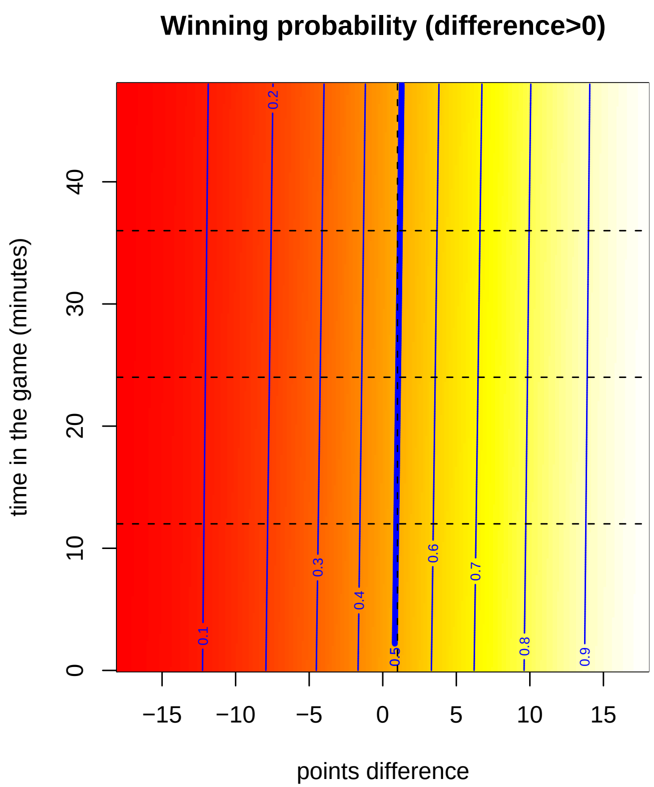
Winning as a function of time and score difference

a natural extention

$$\text{win}_i = \alpha + \varphi((\text{score difference})_{i,t}) + \psi((\text{time in the game})_t) + \varepsilon_i$$

(simple additive model)





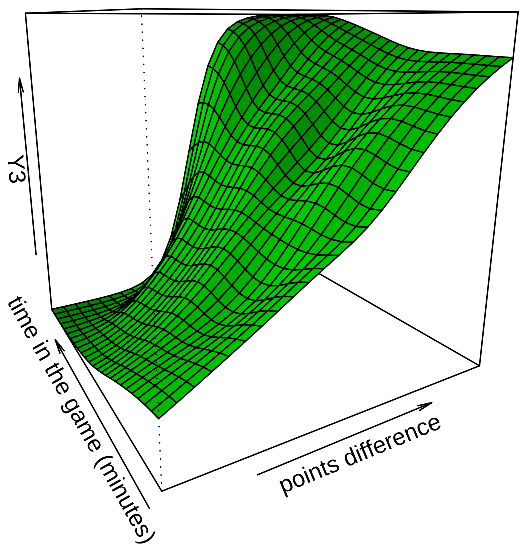
Winning as a function of time and score difference

or more generally

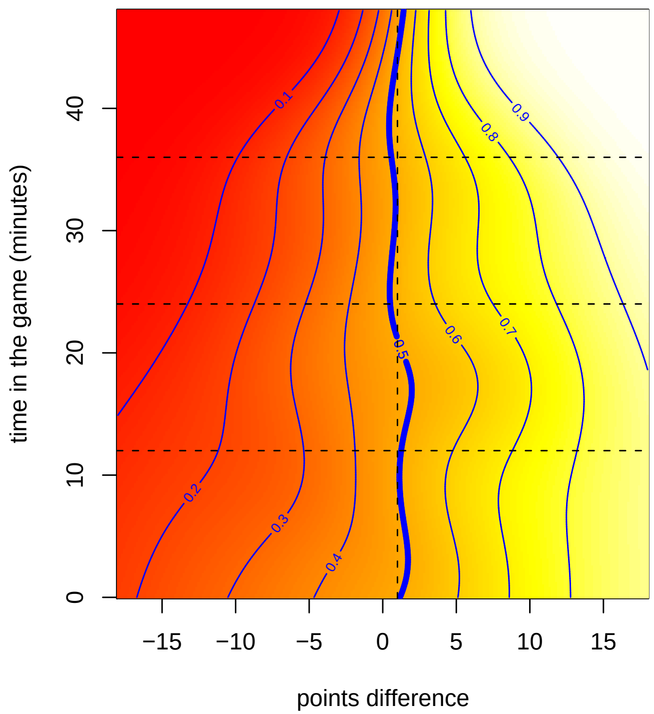
$$\text{win}_i = \alpha + s((\text{score difference})_{i,t}, (\text{time in the game})_t) + \varepsilon_i$$

(functional nonlinear model)

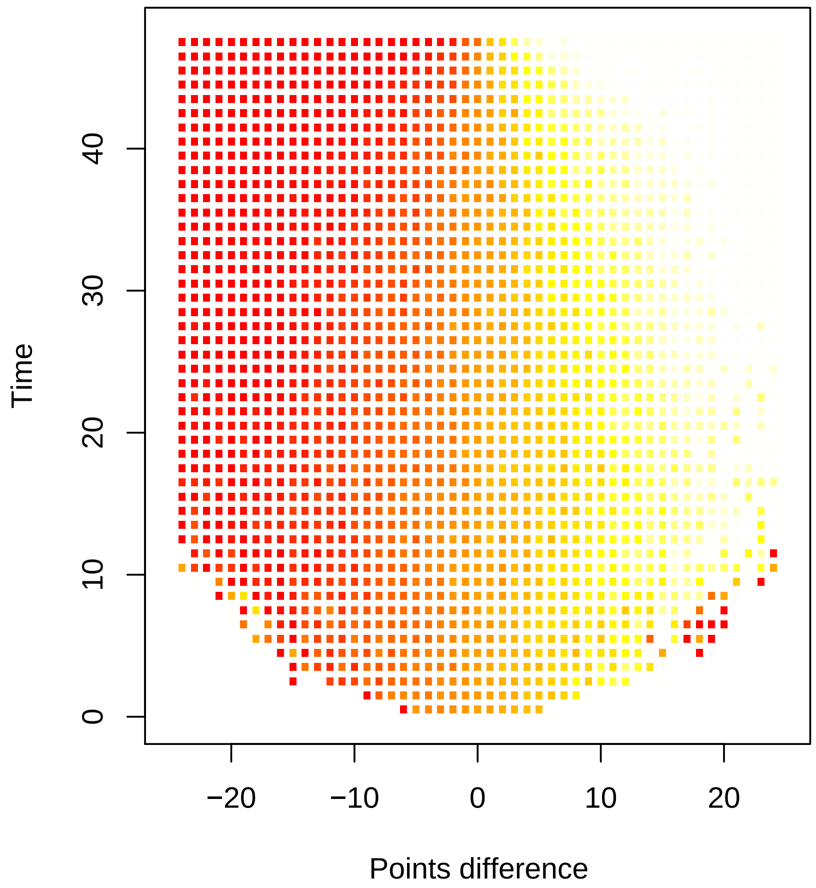
Winning probability (difference>0)

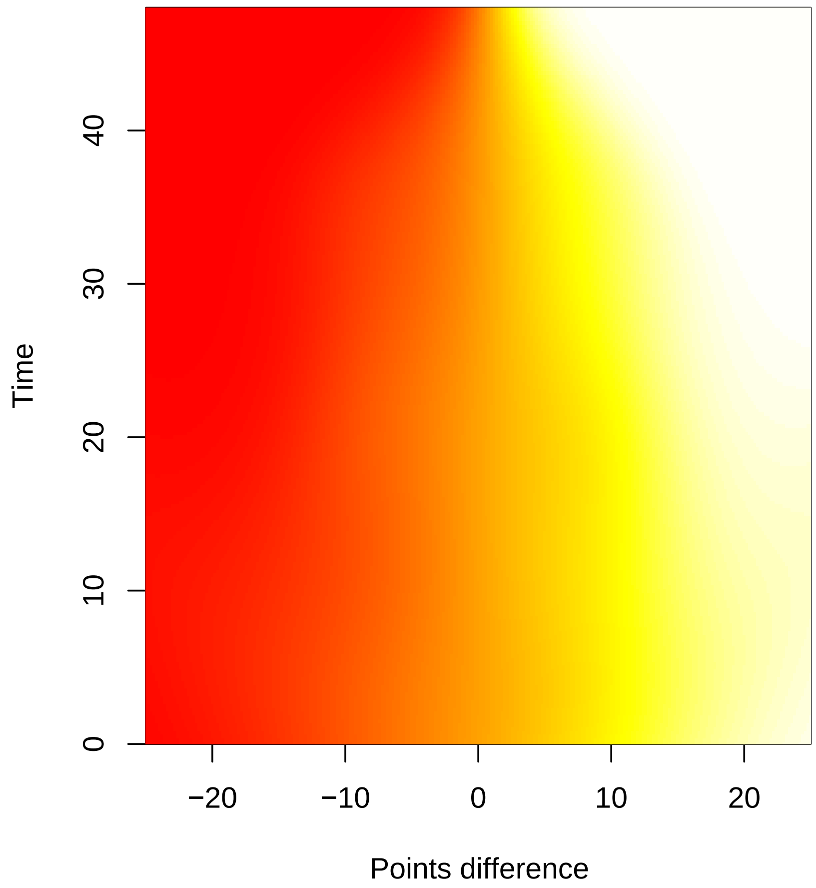


Winning probability (difference>0)

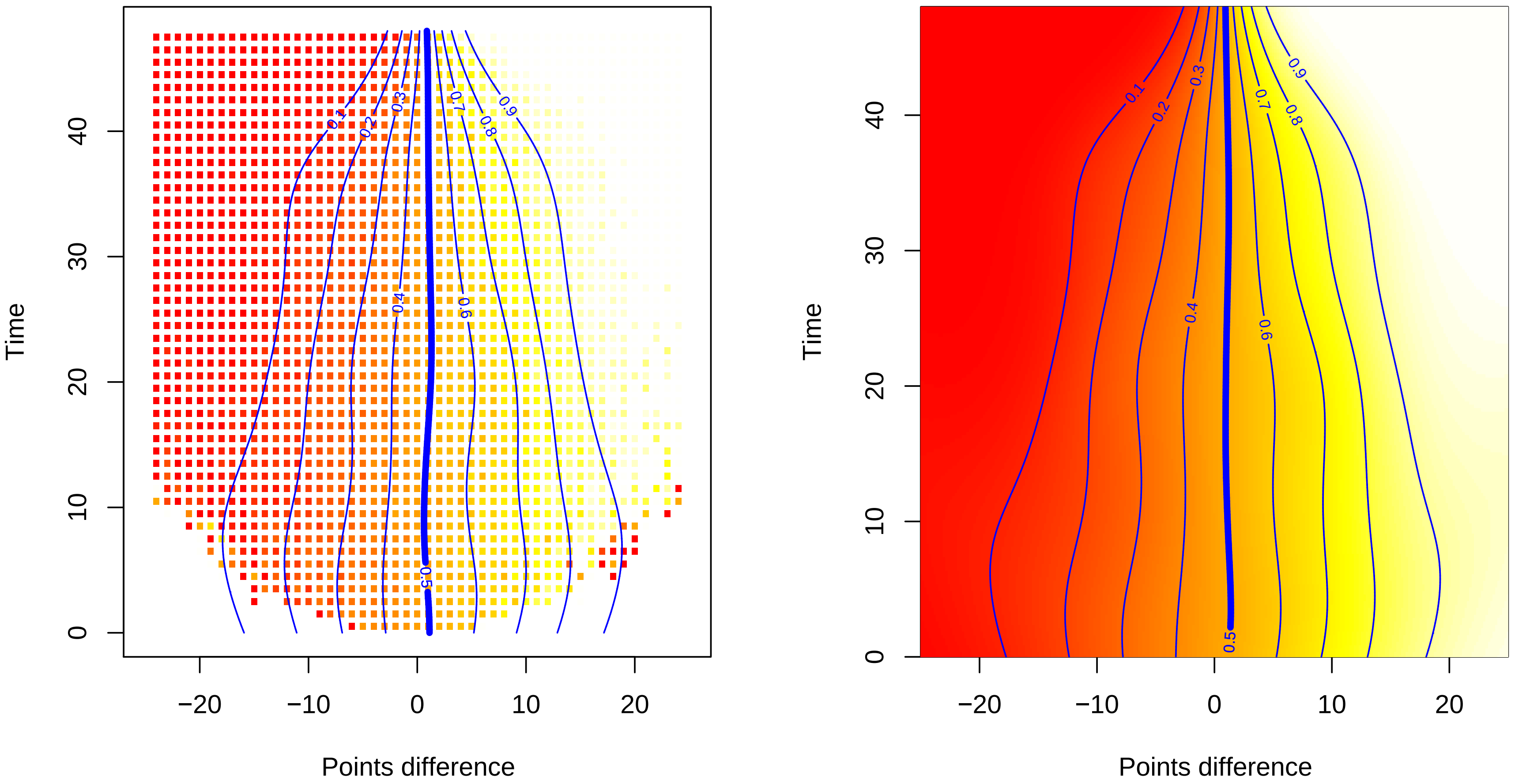


Smooth estimation, versus raw data

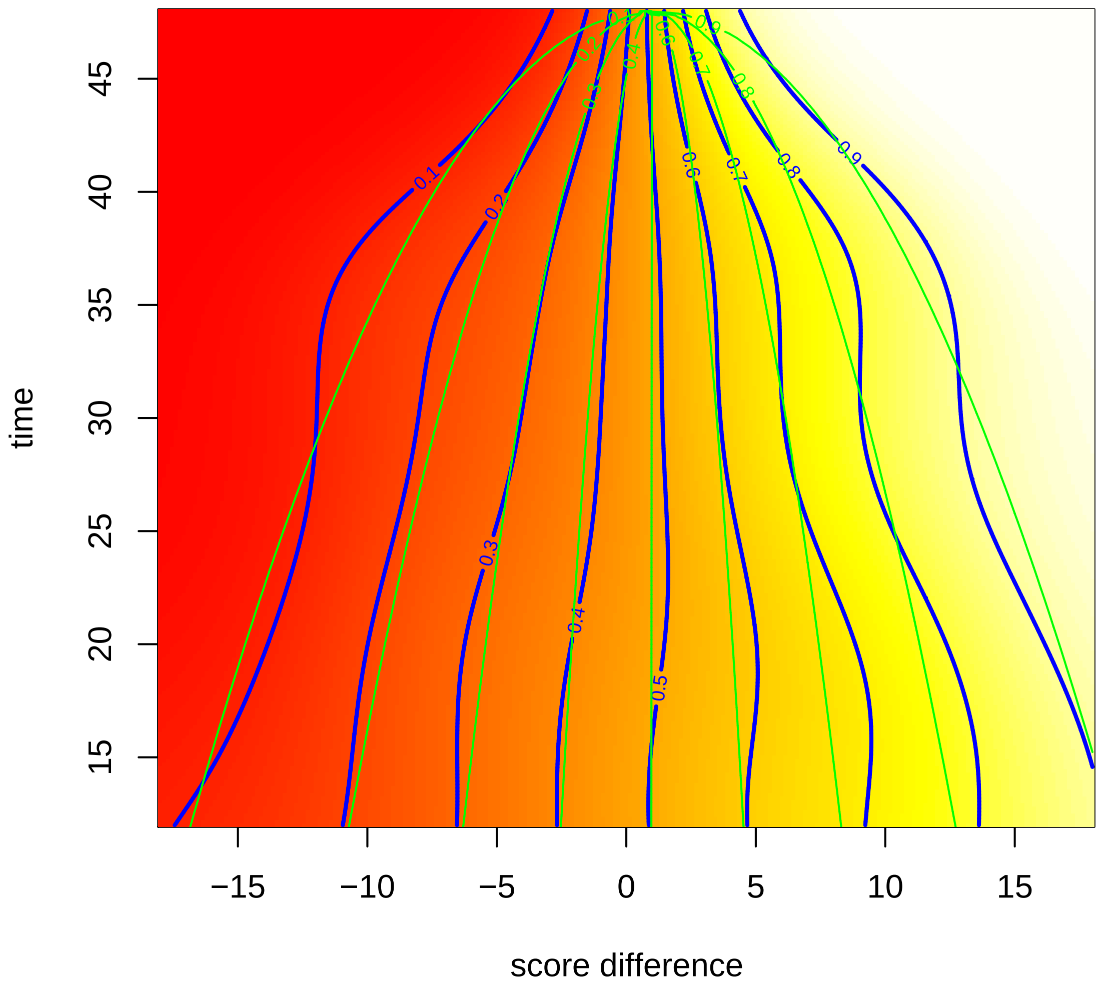




Smooth estimation, versus raw data

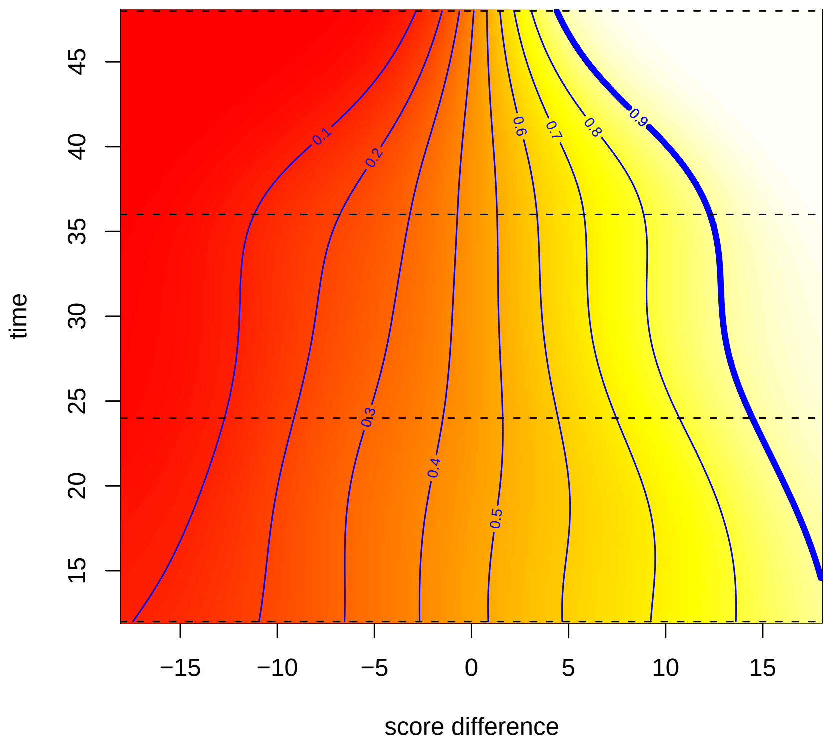


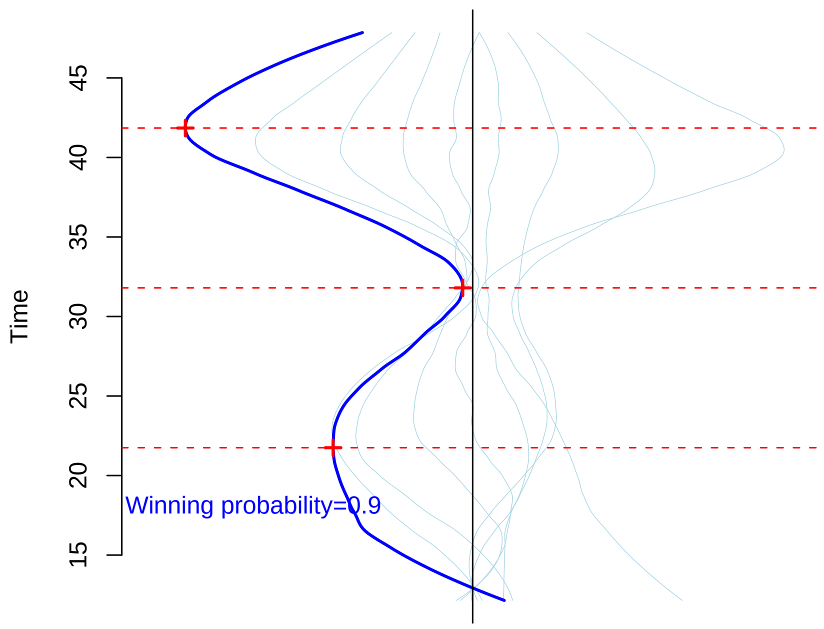
Do teams update their effort ?



When do teams stop their effort ?

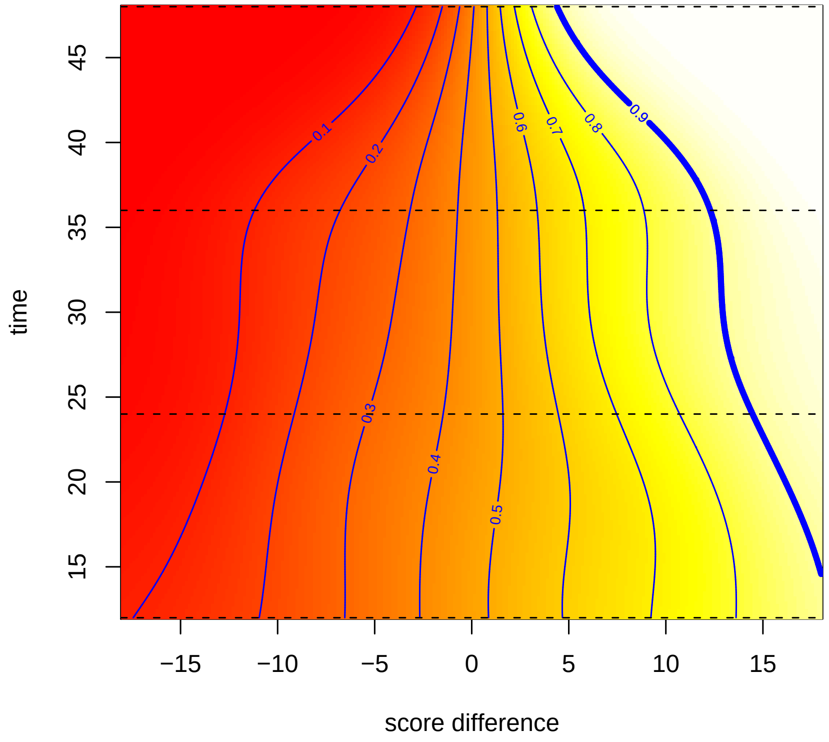
when teams are about to win (90% chance)

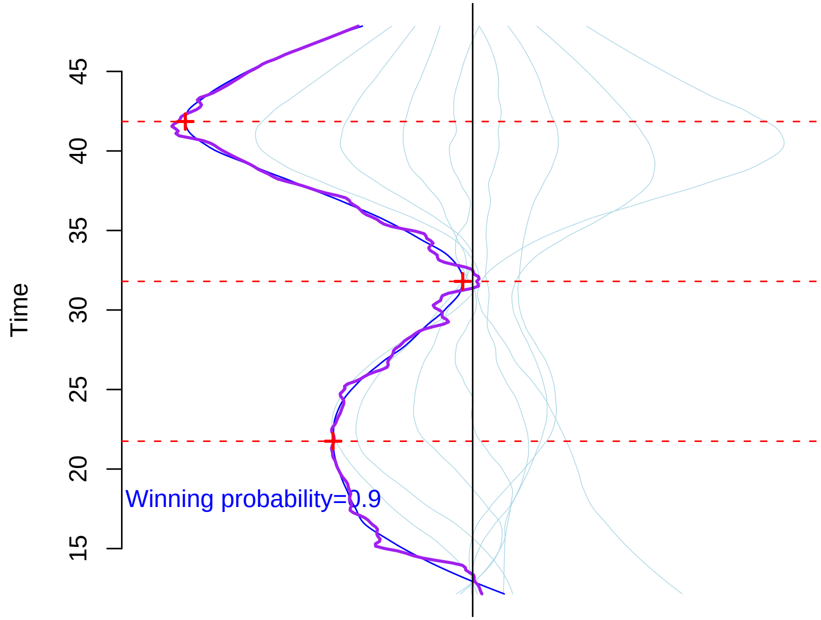




When do teams stop their effort ?

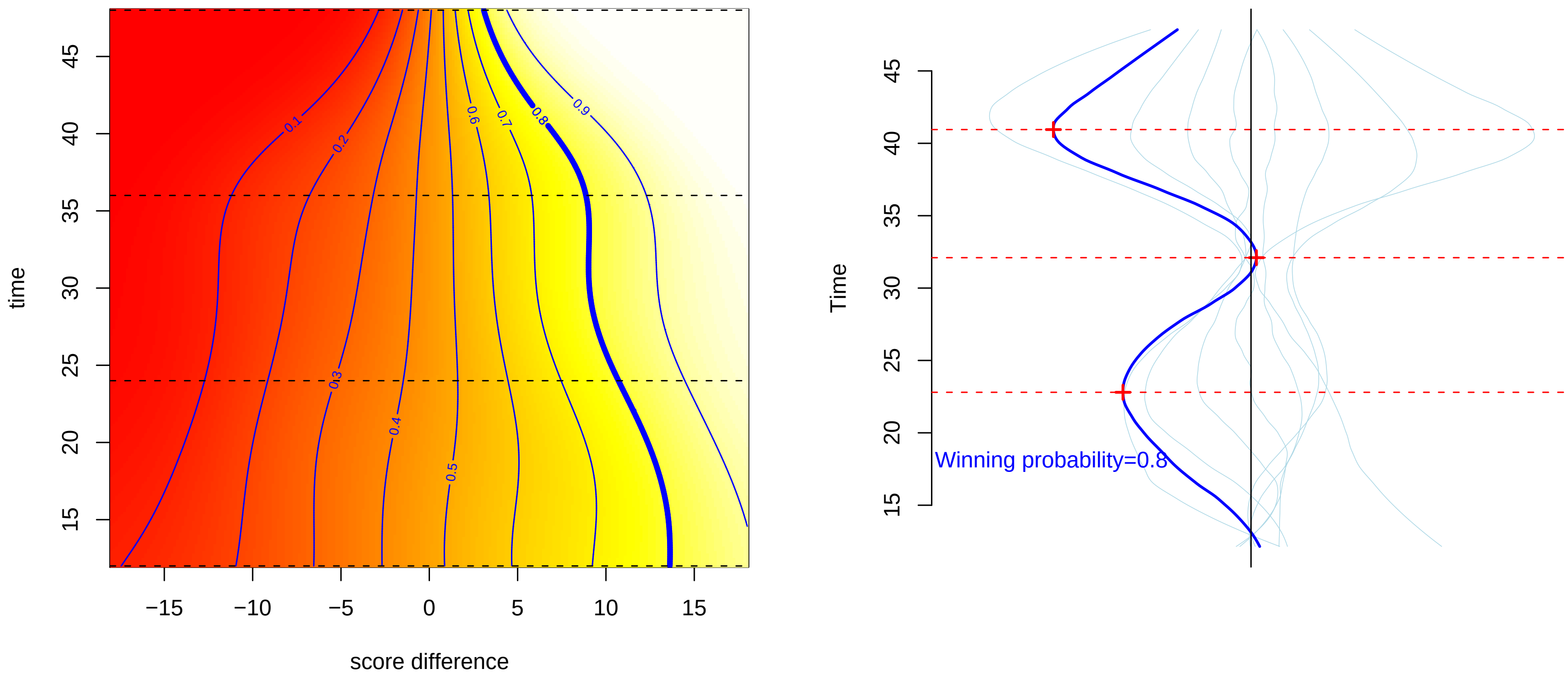
(with a more accurate estimation of the change in the slope)





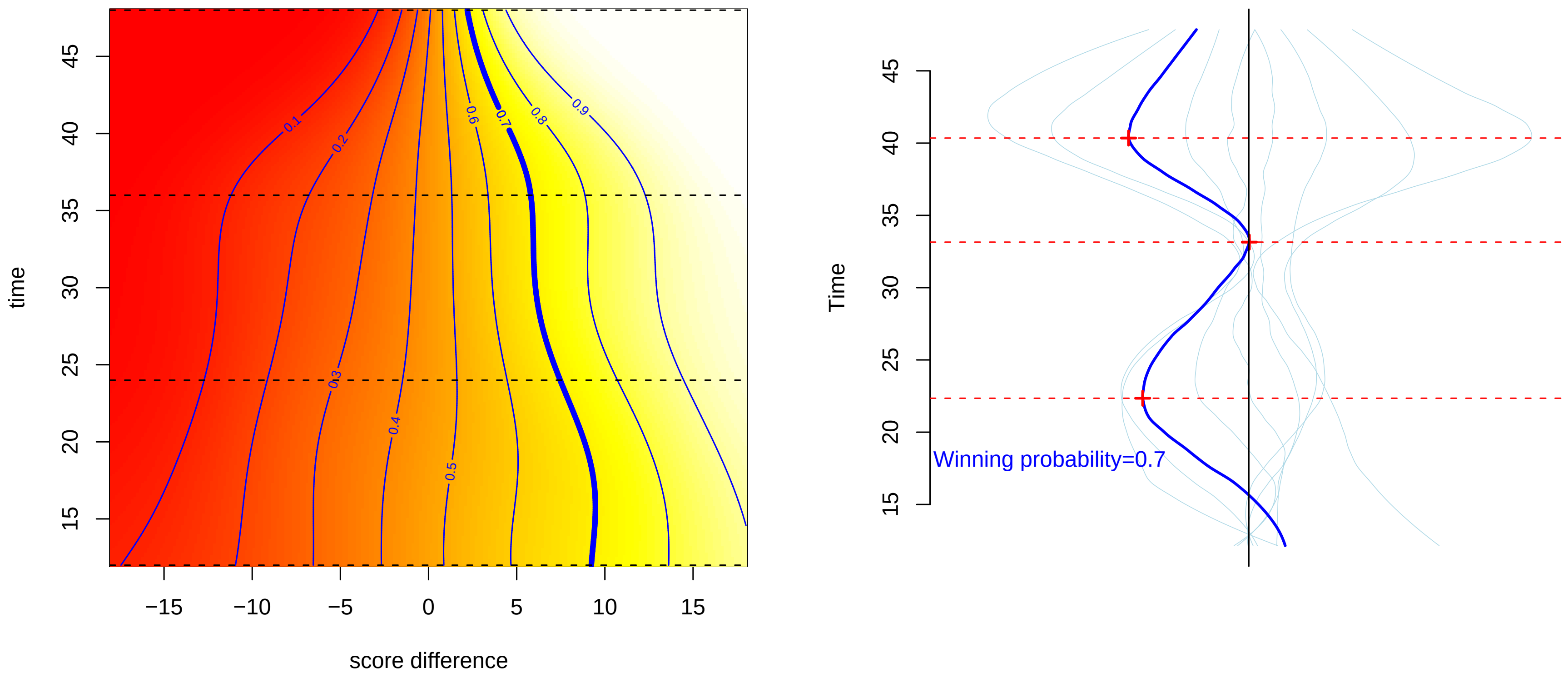
When do teams stop their effort ?

when teams are about to win (80% chance)



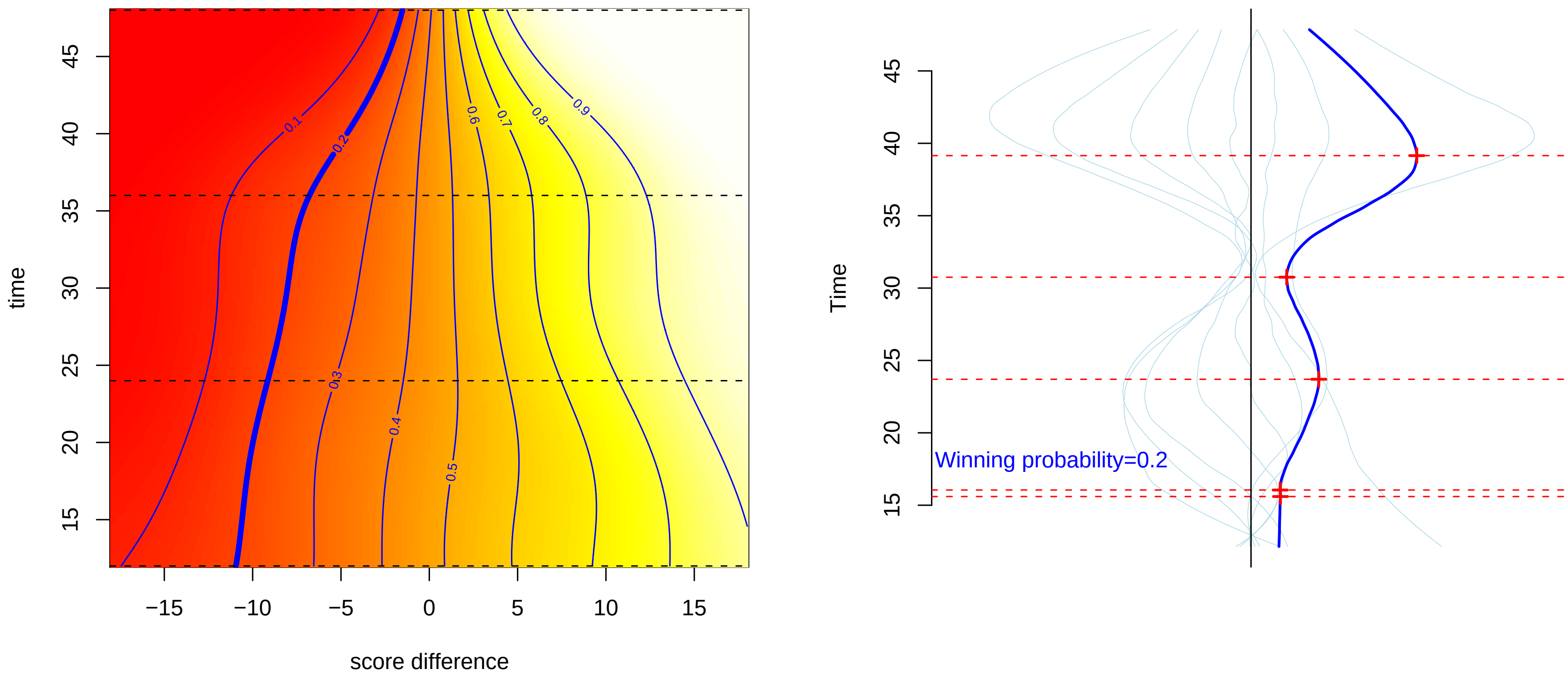
When do teams stop their effort ?

when teams are about to win (70% chance)



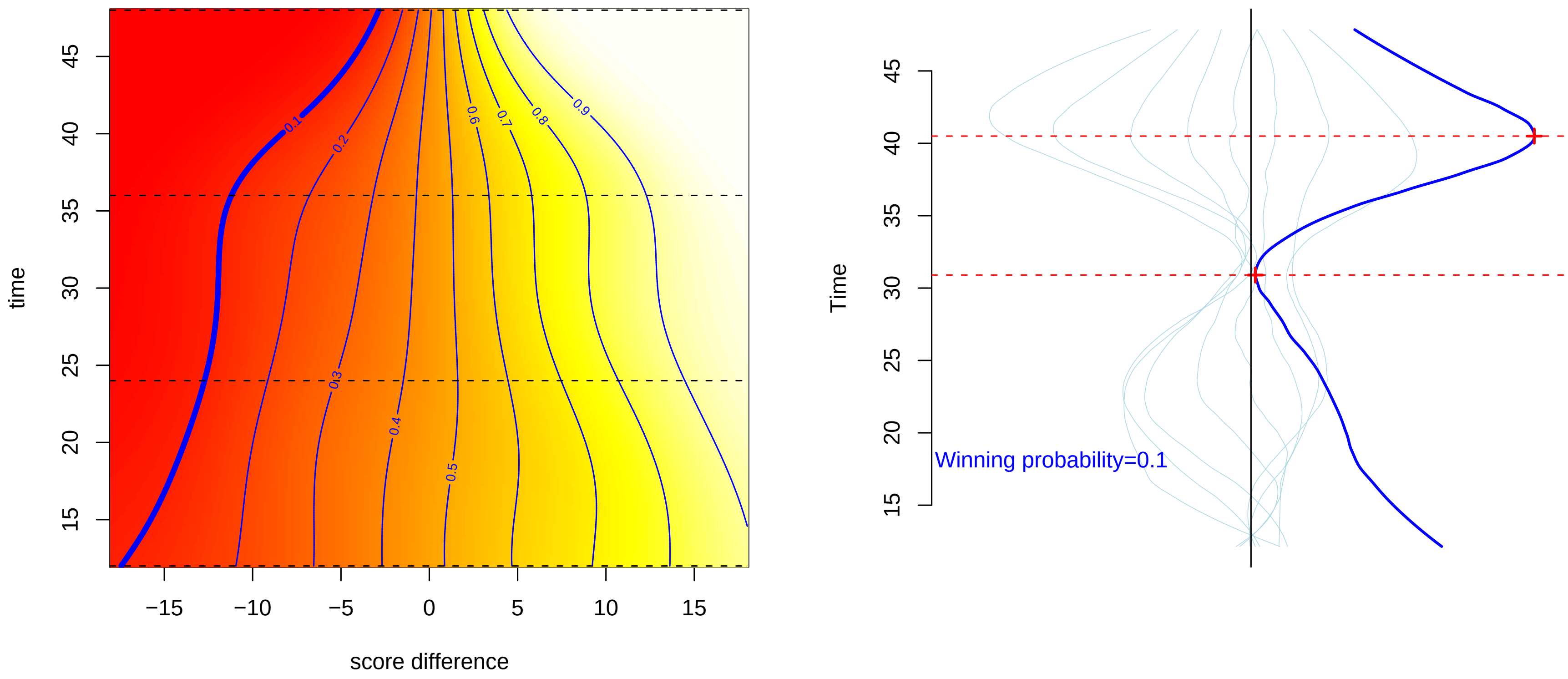
When do teams stop their effort ?

when teams are about to loose (20% chance to win)



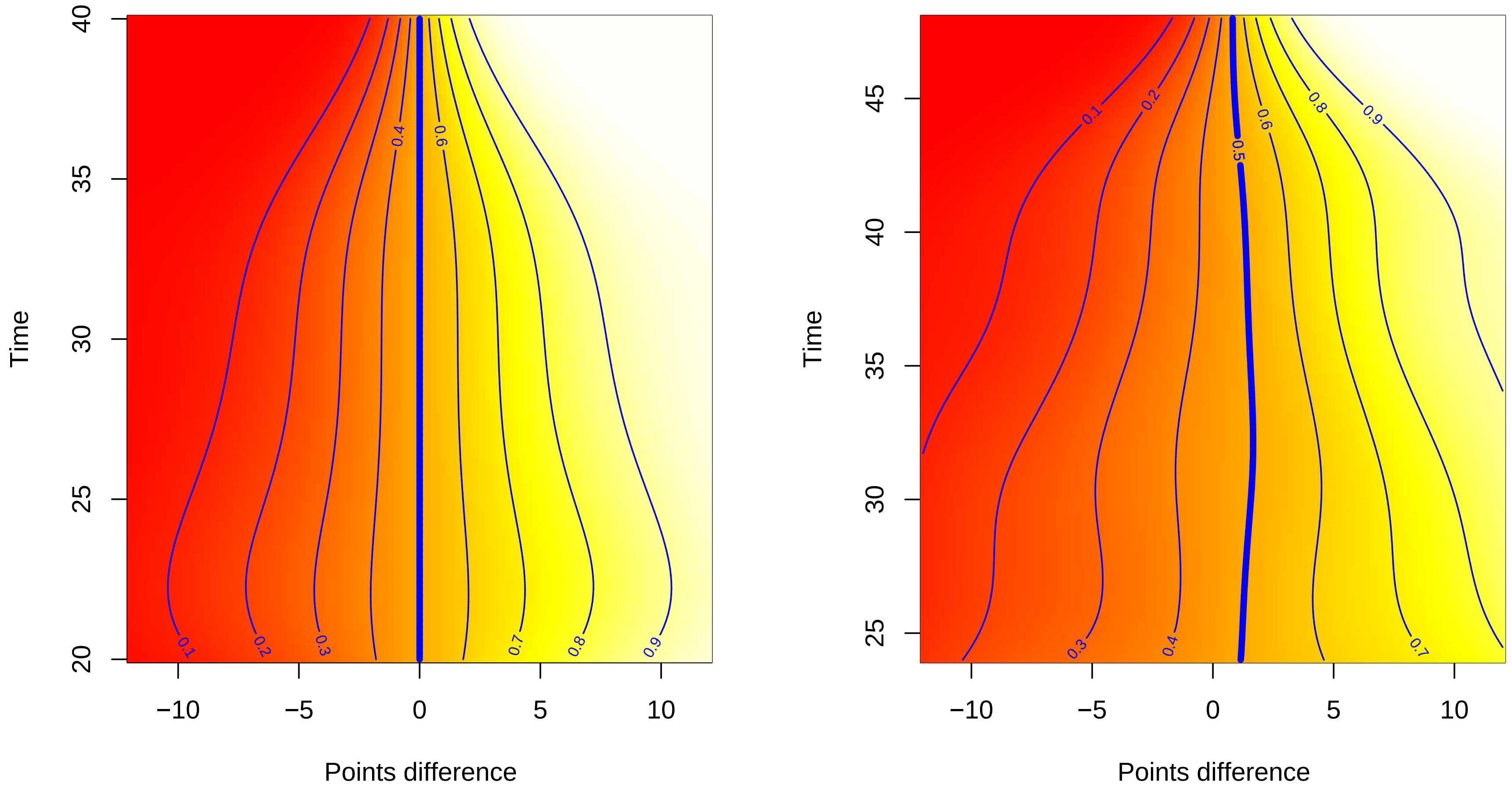
When do teams stop their effort ?

when teams are about to loose (10% chance to win)



NBA players are professionals....

Here are winning probability, college (left) versus NBA (right),



NBA players are professionals....

... when they play at home, college (left) versus NBA (right),