# Modeling heat-waves :
## return period for non-stationary extremes

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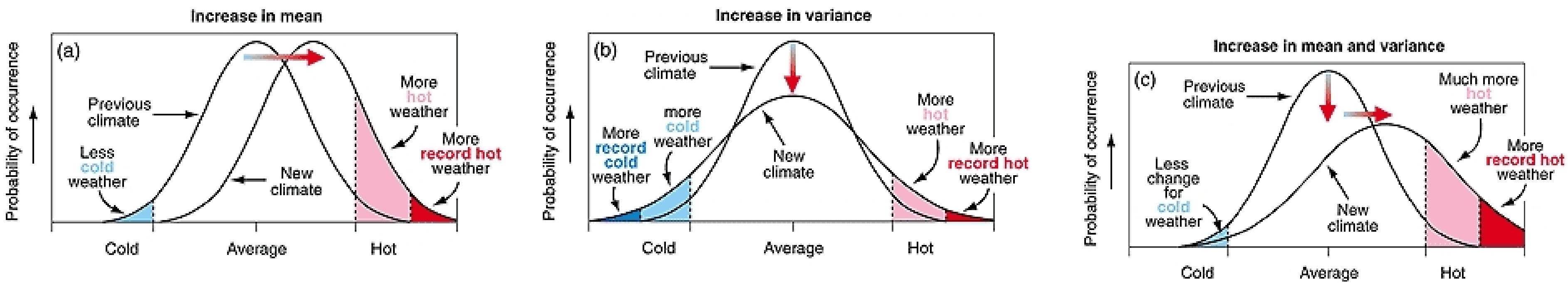
# Motivation

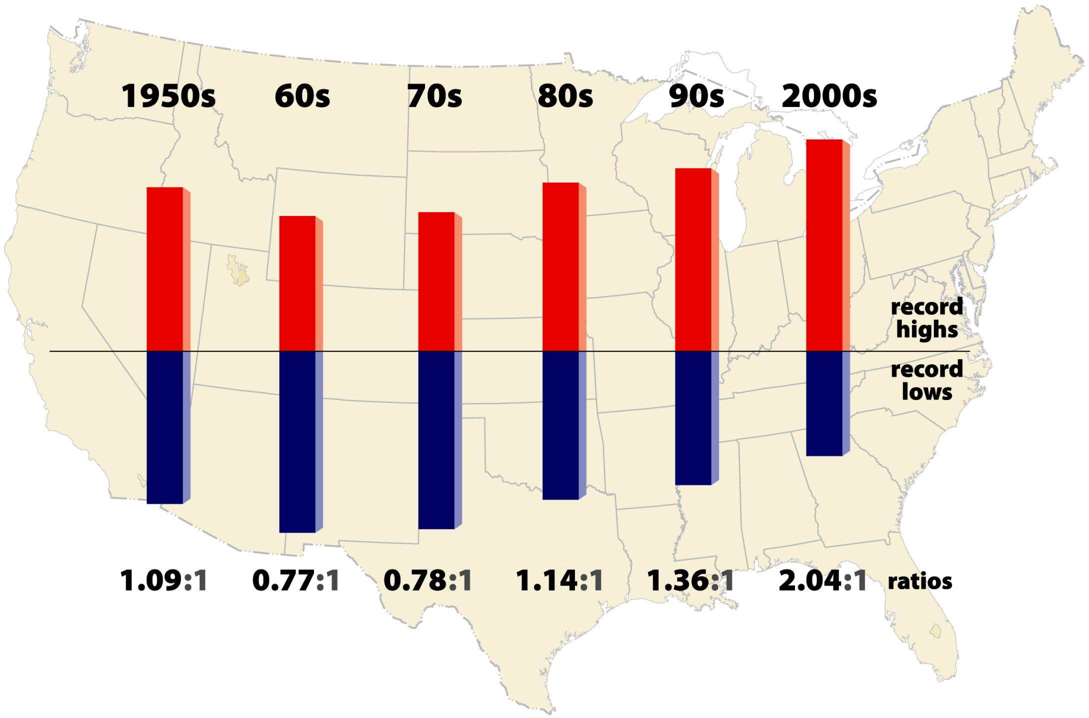
"*Tous nos scénarios à dix ans montrent une aggravation des tempêtes, inondations, sécheresses. Les catastrophes centennales vont devenir plus fréquentes.*" Jean-Marc Lamère, délégué général de la Fédération Française des Sociétés d'Assurances, 2003.

Climate *change*, from Third IPCC Agreement ,2001 :



# Motivation

"*there is no longer any doubt that the Earth's climate is changing* [...] *globally, nine of the past 10 years have been the warmest since records began in* 1861", February 2005, opening the conference *Climate change : a global, national and regional challenge*, Dennis Tirpak.




source : Meehkl*et al.* (2009).

## The European heatwave of 2003

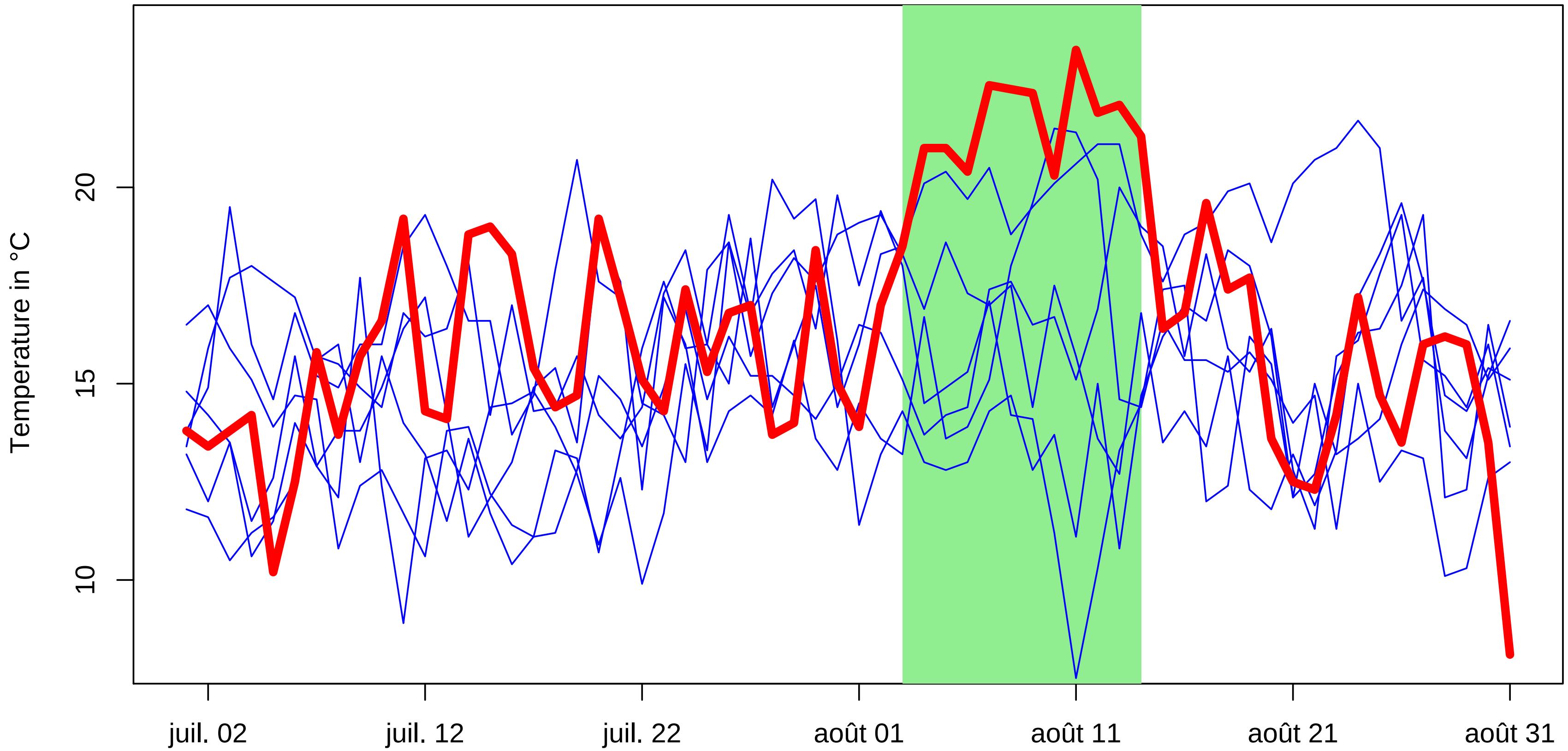
Third IPCC Assessment, 2001 : treatment of extremes (e.g. trends in extreme high temperature) is "*clearly inadequate*". KARL & TRENBERTH (2003) noticed that "*the likely outcome is more frequent heat waves*", "*more intense and longer lasting*" added MEEHL & TEBALDI (2004).

In Nîmes, there were more than 30 days with temperatures higher than 35° C (versus 4 in hot summers, and 12 in the previous heat wave, in 1947).

Similarly, the average maximum (minimum) temperature in Paris peaked over 35° C for 10 consecutive days, on 4-13 August. Previous records were 4 days in 1998 (8 to 11 of August), and 5 days in 1911 (8 to 12 of August).

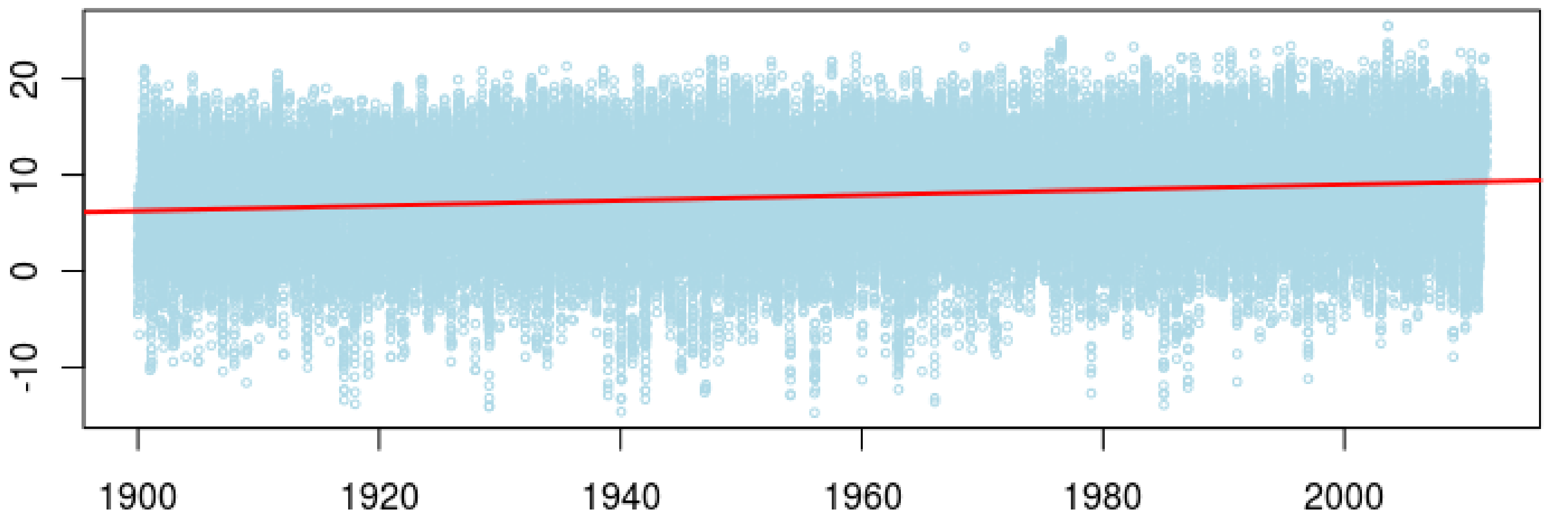
Similar conditions were found in London, where maximum temperatures peaked above 30°C during the period 4-13 August

(see e.g. BURT (2004), BURT & EDEN (2004) and FINK *et al.* (2004).)


Temperature in °C
10
15
20
juil. 02
juil. 12
juil. 22
août 01
août 11
août 21
août 31

## Which *temperature* might be interesting ?

Karl & Knight (1997) , modeling of the 1995 heatwave in Chicago : minimum temperature should be most important for health impact (see also Kovats & Koppe (2005)), several nights with no relief from very warm nighttime



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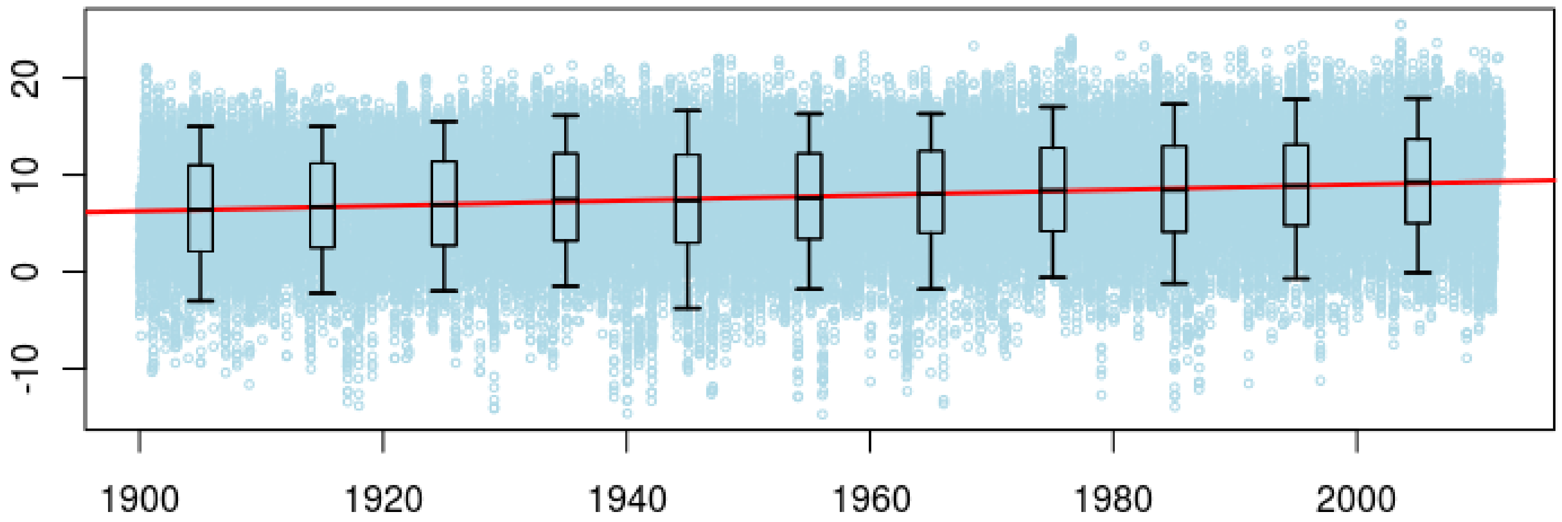


## Modeling temperature

Consider the following decomposition

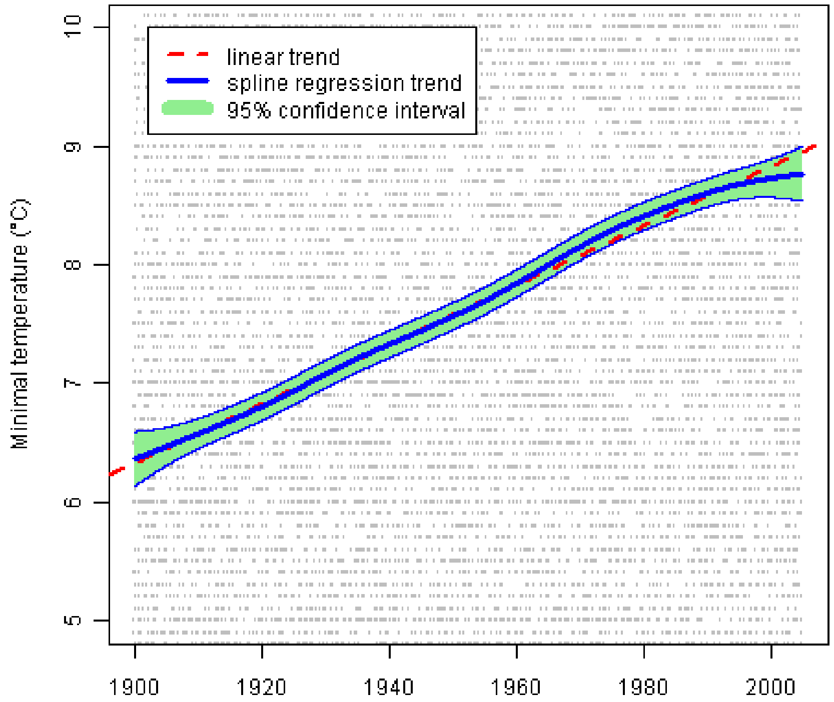
$$Y_t = \mu_t + S_t + X_t$$

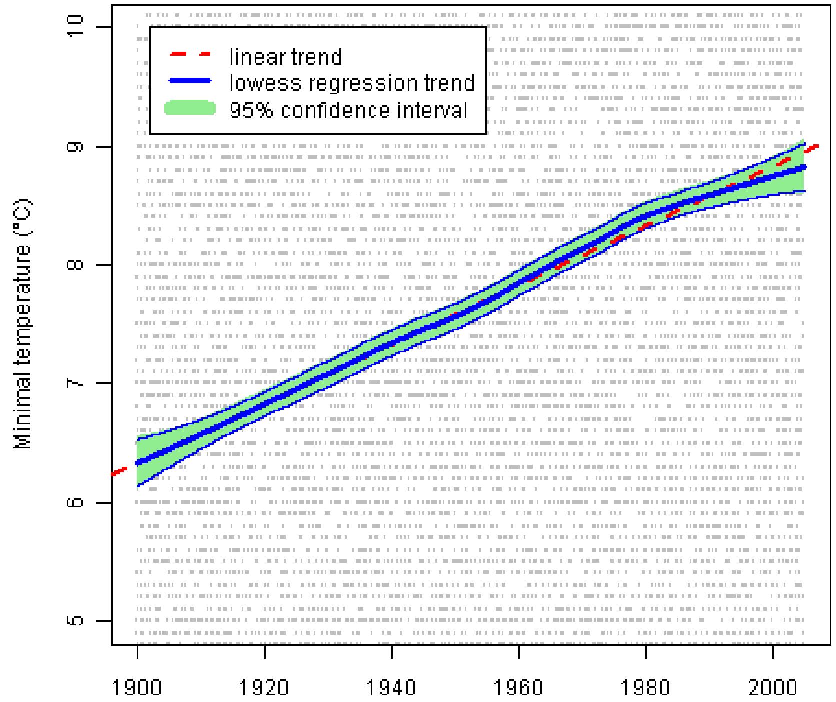
where

- $\mu_t$ is a (linear) general tendency
- $S_t$ is a seasonal cycle
- $X_t$ is the remaining (stationary) noise

# Nonstationarity and *linear* trend

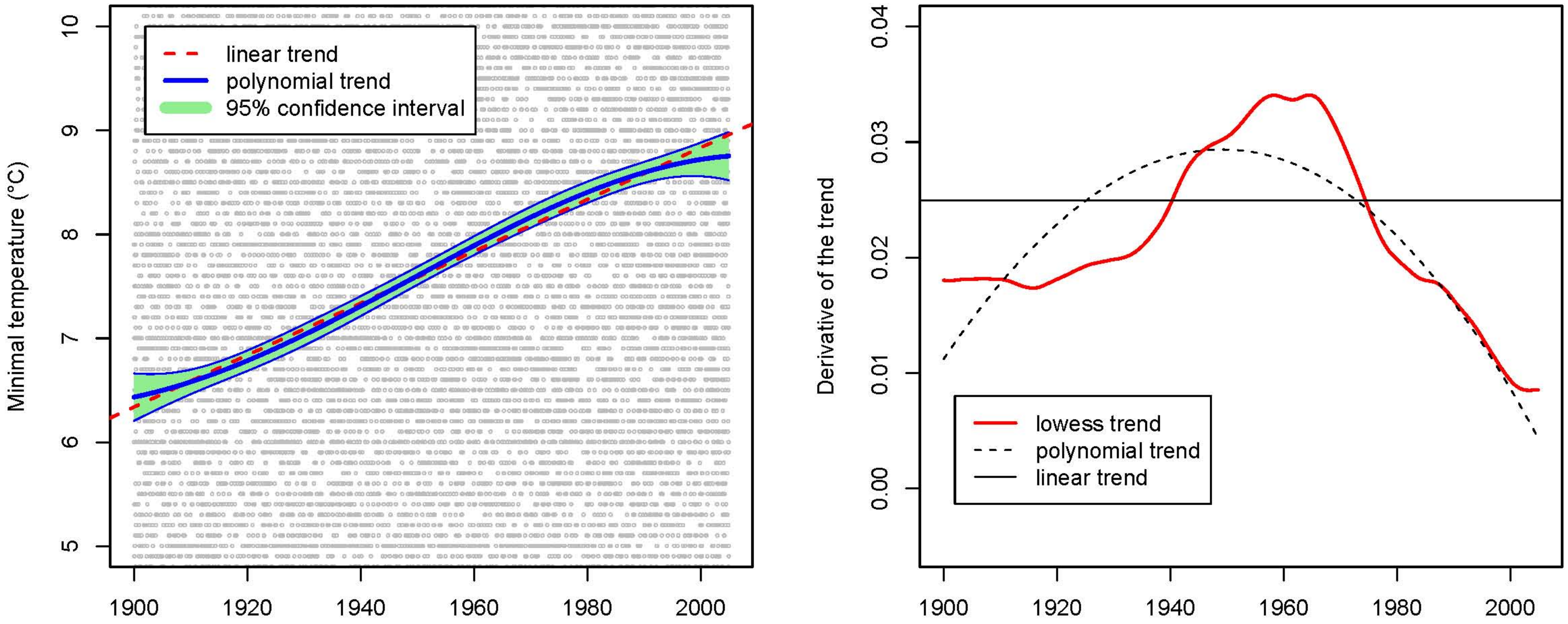
Consider a spline and lowess regression



# Nonstationarity and *linear* trend

or a polynomial regression,and compare local slopes,



## Nonstationarity and *linear* trend

Several authors (from Lane*et al.* (1994) to Black*et al.* (2004)) have tried to explain global warming, and to find explanatory factors.

As pointed out in Quereda*et al.* (2000), the "*analysis of the trend is difficult and could be biased by non-climatic processes such as the urban effect*". In fact, "*most of the temperature rise could be due to an urban effect*" : global warming can be understood as one of the consequence of "*global pollution*" (see also Houghton (1997) or Braun*et al.* (2004) for a detailed study of the impact of transportation).

## Quantile regression to describe 'extremal' temperature

See Yan (2002), Meehl & Tebaldi (2004), or Alexander*et al.* (2006).

Least square used to estimate a linear model for $\mathbb{E}(Y_t)$,

$$\min_{\beta_0,\beta_1}\sum_{t=1}^{T}(Y_t-[\beta_0+\beta_1 t])^2=\min_{\beta_0,\beta_1}\sum_{t=1}^{T}\mathcal{R}(Y_t-[\beta_0+\beta_1 t])$$

where $\mathcal{R}(x)=x^2$.

Quantile regression can be used, let $p\in(0,1)$, and consider

$$\min_{\beta_0,\beta_1}\sum_{t=1}^{T}\mathcal{R}(Y_t-[\beta_0+\beta_1 t])$$

where

$$\mathcal{R}_p(x)=x\cdot(p-\mathbf{1}(x<0))=\left\{\begin{array}{l} x\cdot(p-1)\text{ if }x<0\\ x\cdot p\text{ if }x>0\end{array}\right.$$

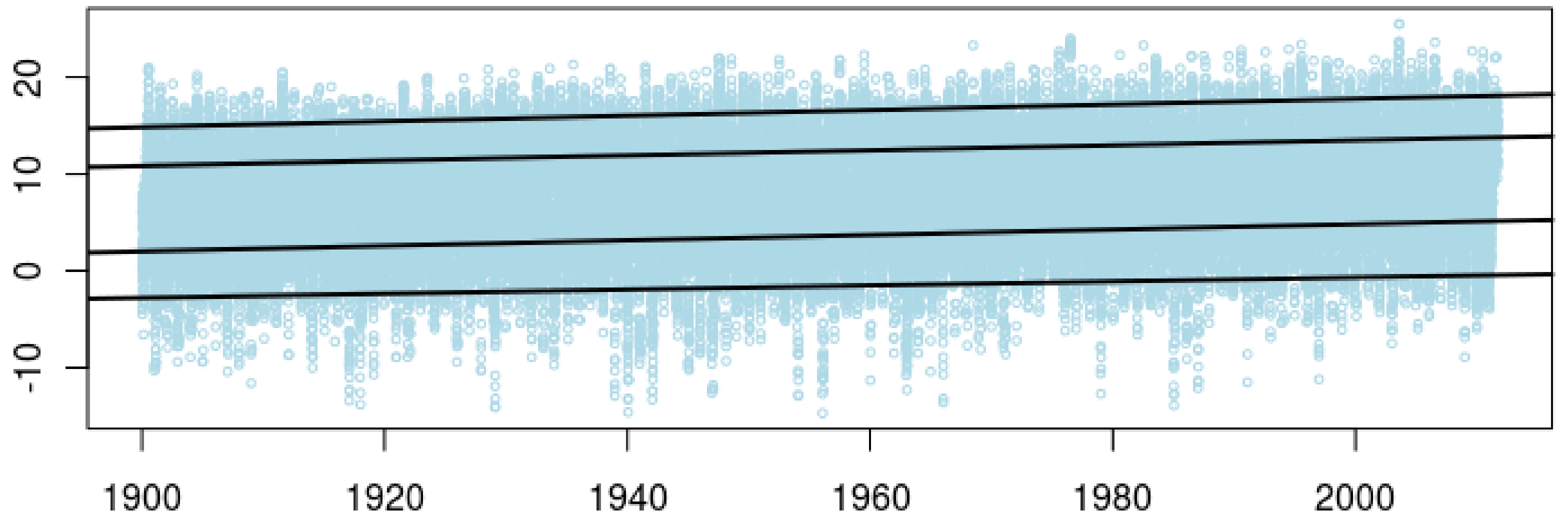
**Remark** :

$$\mathbb{E}(Y) = \text{argmin}\left\{\mathbb{E}\left((Y-\gamma)^2\right)\right\} = \text{argmin}\left\{\int (Y-\gamma)^2 dF(y)\right\}$$

while

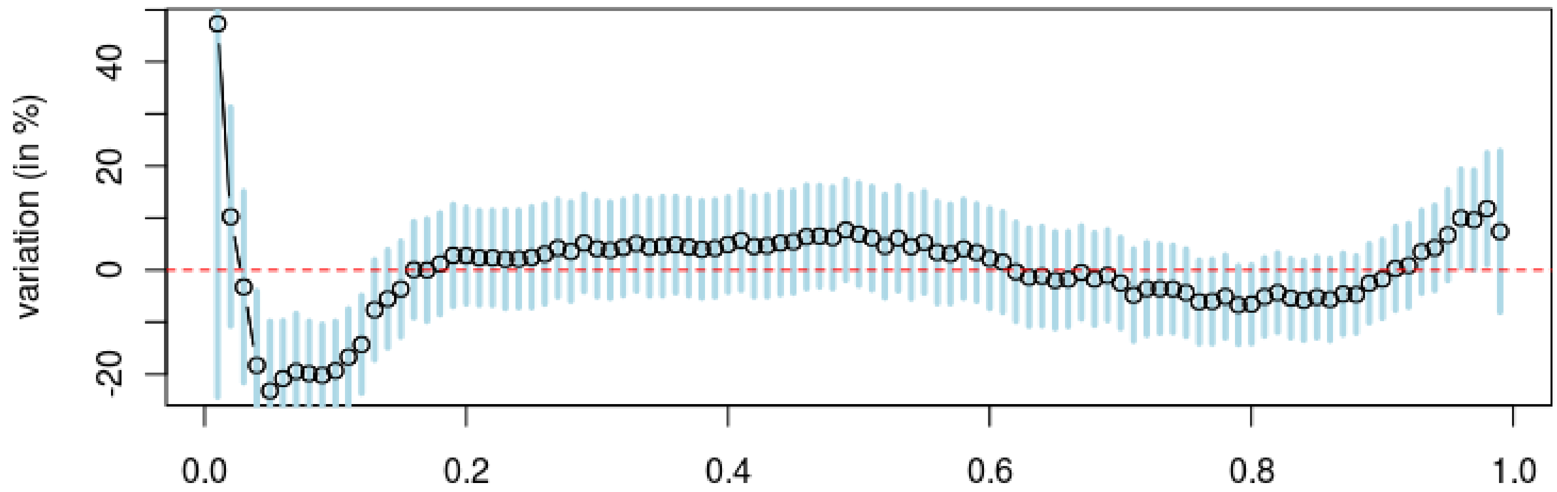
$$Q_p(Y) = \text{argmin}\left\{[p-1]\cdot\int_{-\infty}^{\gamma}[\gamma - x]dF(x) + p\cdot\int_{\gamma}^{+\infty}[x-\gamma]dF(x)\right\}.$$

5%, 25%, 75%, and 95% quantile regressions,

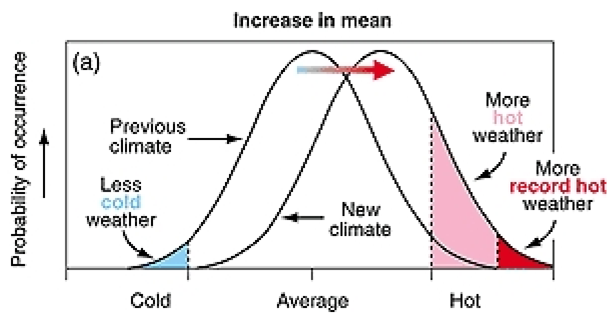


# Quantile regression to describe 'extremal' temperature

slopes ($\beta_1$) of quantile regressions, as functions of probability $q \in (0,1)$,

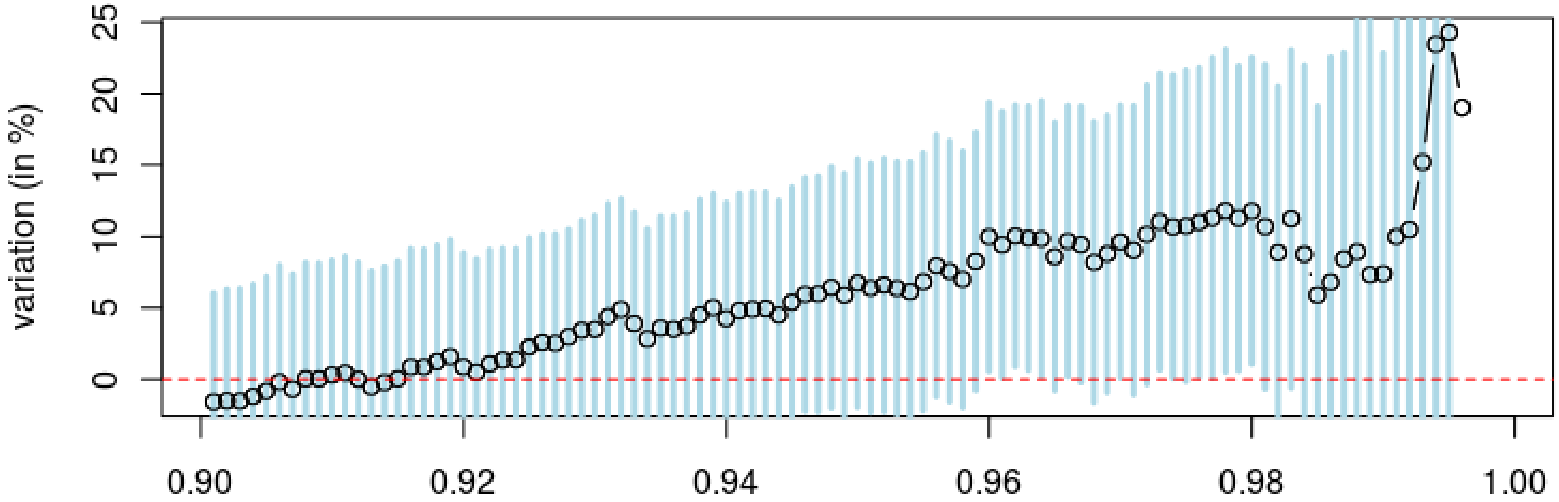



**Remark** : constant rate means that scenario 1 is realistic in the Third IPCC Agreement, 2001 :



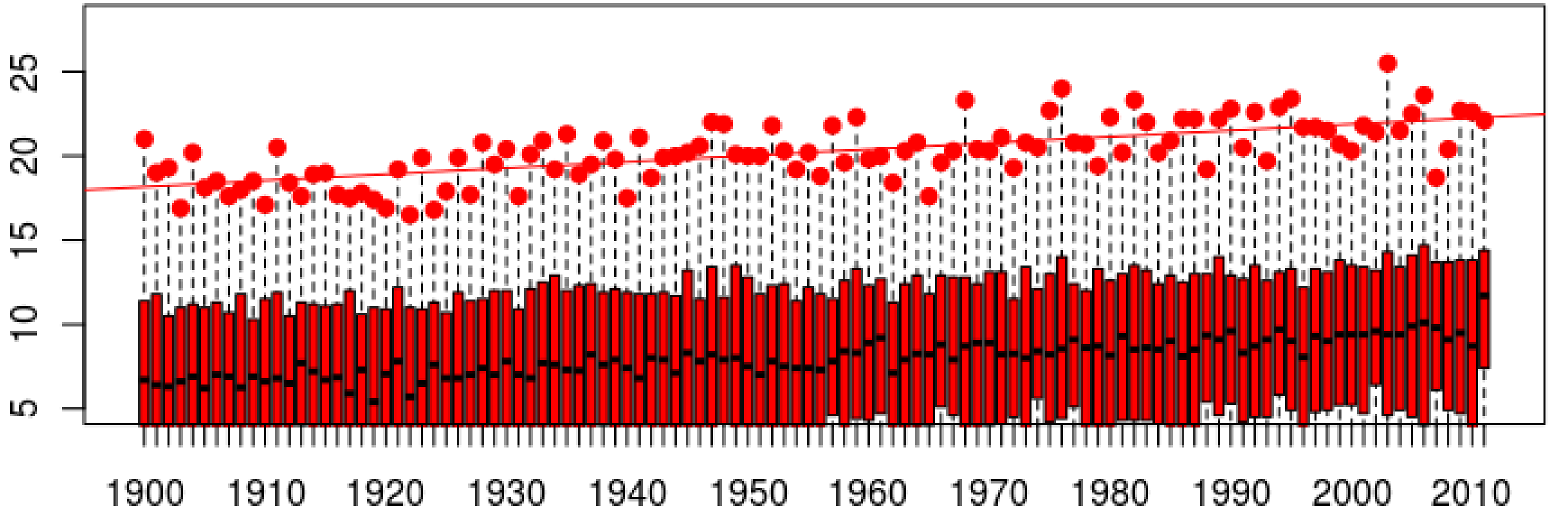
## Quantile regression to describe 'extremal' temperature

slopes ($\beta_1$) of quantile regressions, as functions of probability $q \in (0.9, 1)$,



# Regression for yearly maxima

Let $M_i$ denote the highest minimal daily temperature observed during year $i$,



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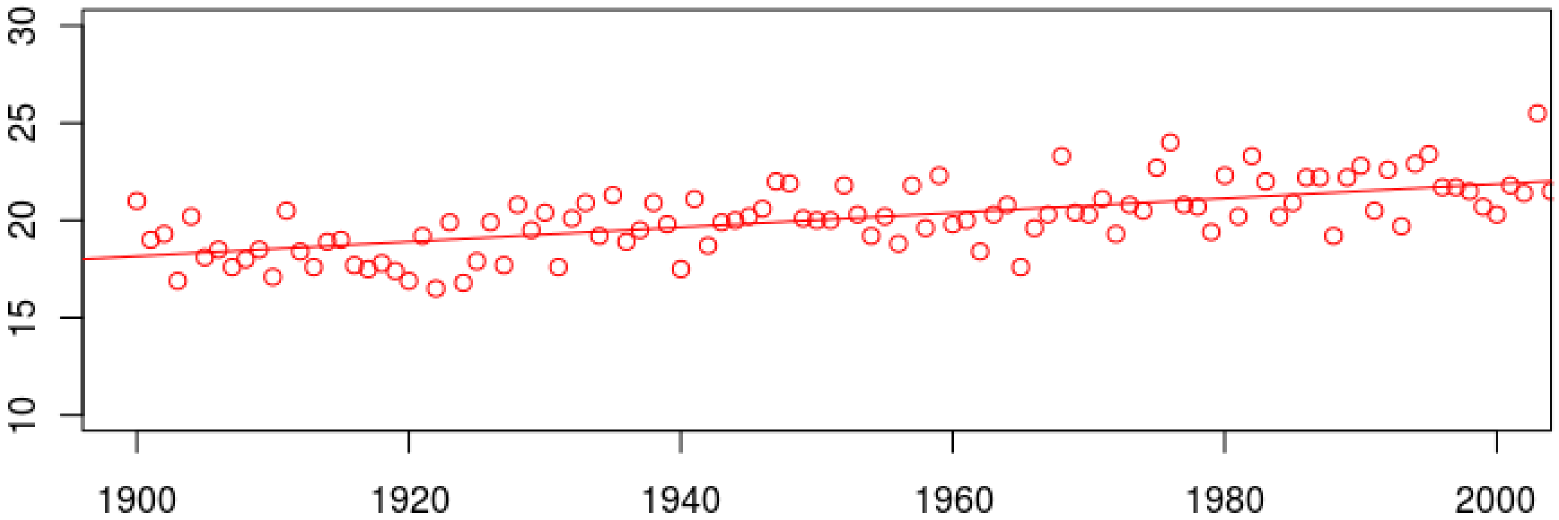


## Linear trend, and Gaussian noise

BENESTAD (2003) or REDNER & PETERSEN (2006)

temperature for a given (calendar) day is an "*independent Gaussian random variable with constant standard deviation* $\sigma$ *and a mean that increases at constant speed* $\nu$"

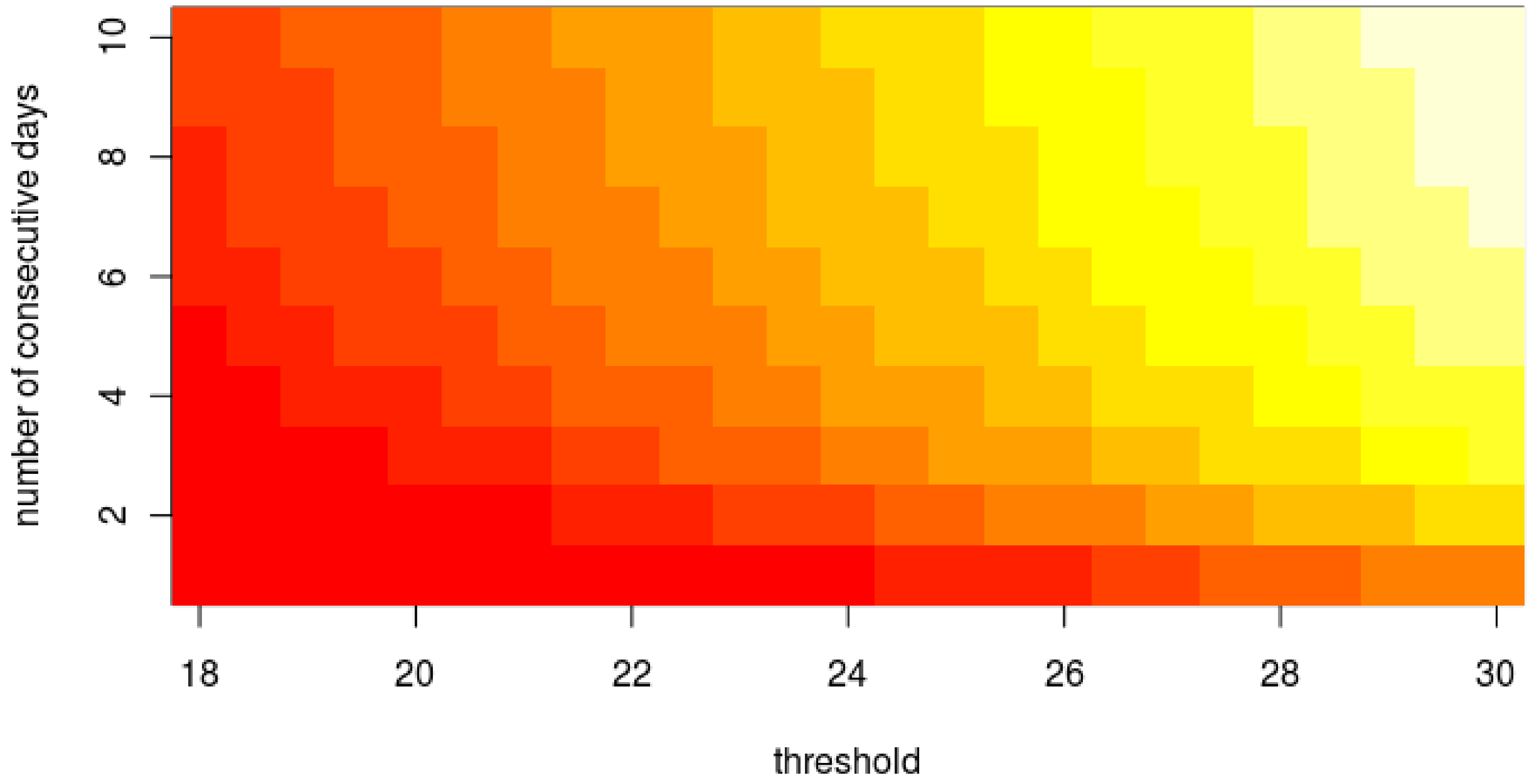
In the US, $\nu = 0.03°$ C per year, and $\sigma = 3.5°$ C

In Paris, $\nu = 0.027°$ C per year, and $\sigma = 3.23°$ C

Assuming independence it is possible to estimate return periods

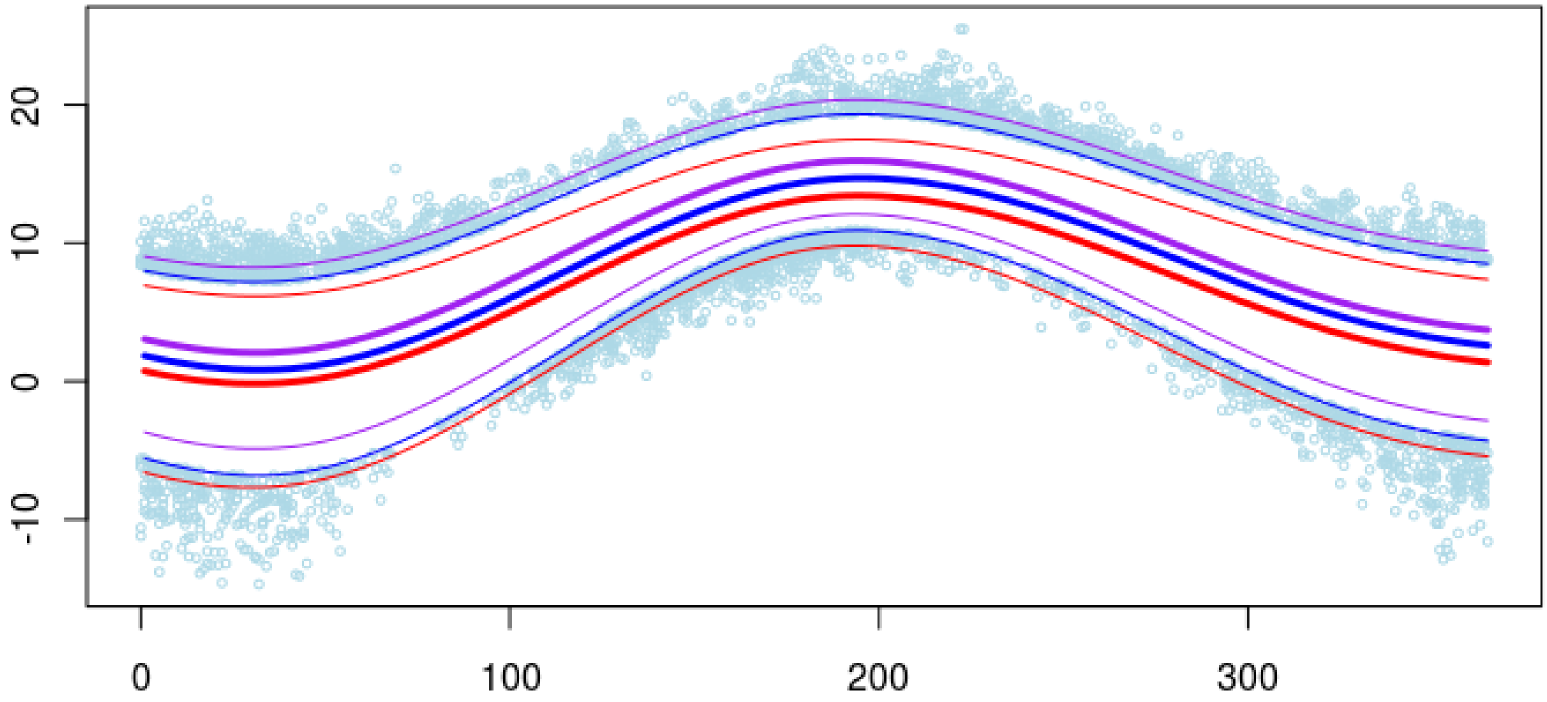
# Linear trend, and Gaussian noise

Return period of $k$ consecutive days with temperature exceeding $s$,



## The seasonal component

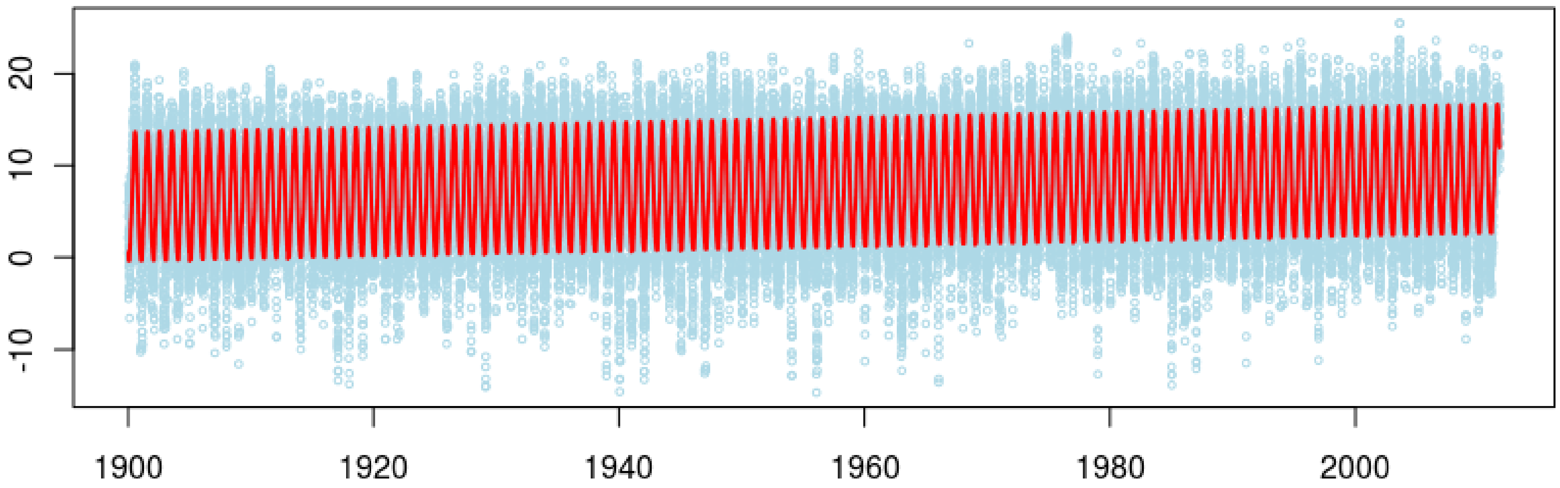
Seasonal pattern during the yearly




did not use a cosine function to model $S_t$ but a spline regression (on circular data),

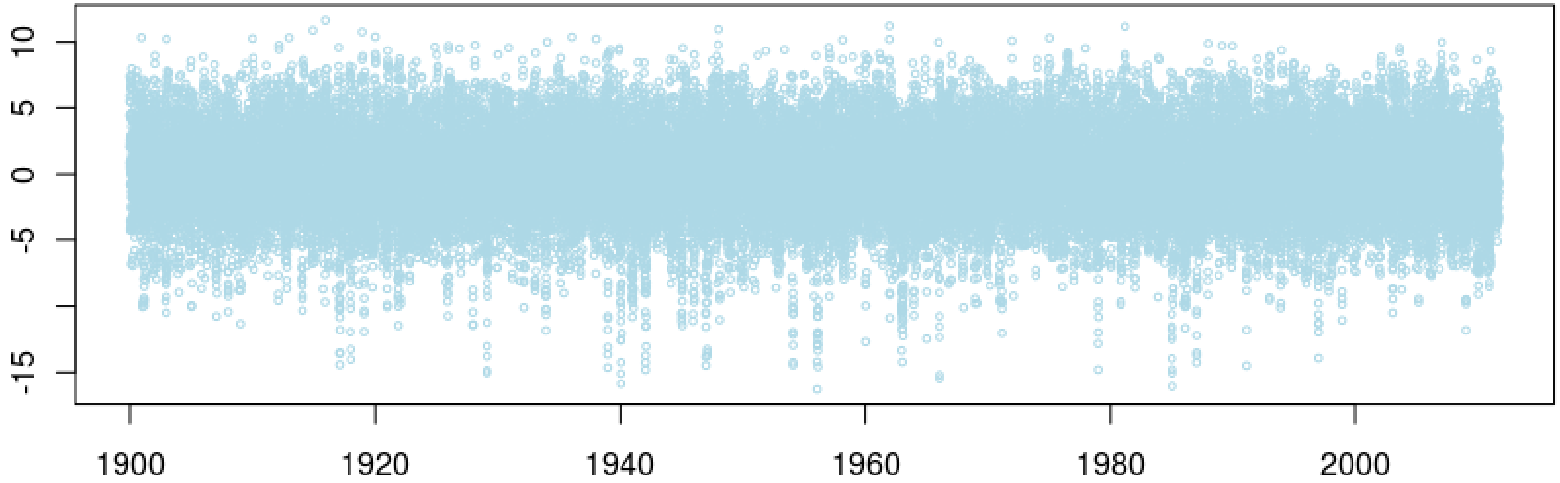
## The residual part (or stationary component)

Let $\widehat{X}_t = Y_t - \left(\widehat{\beta}_0 + \widehat{\beta}_1 t + \widehat{S}_t\right)$



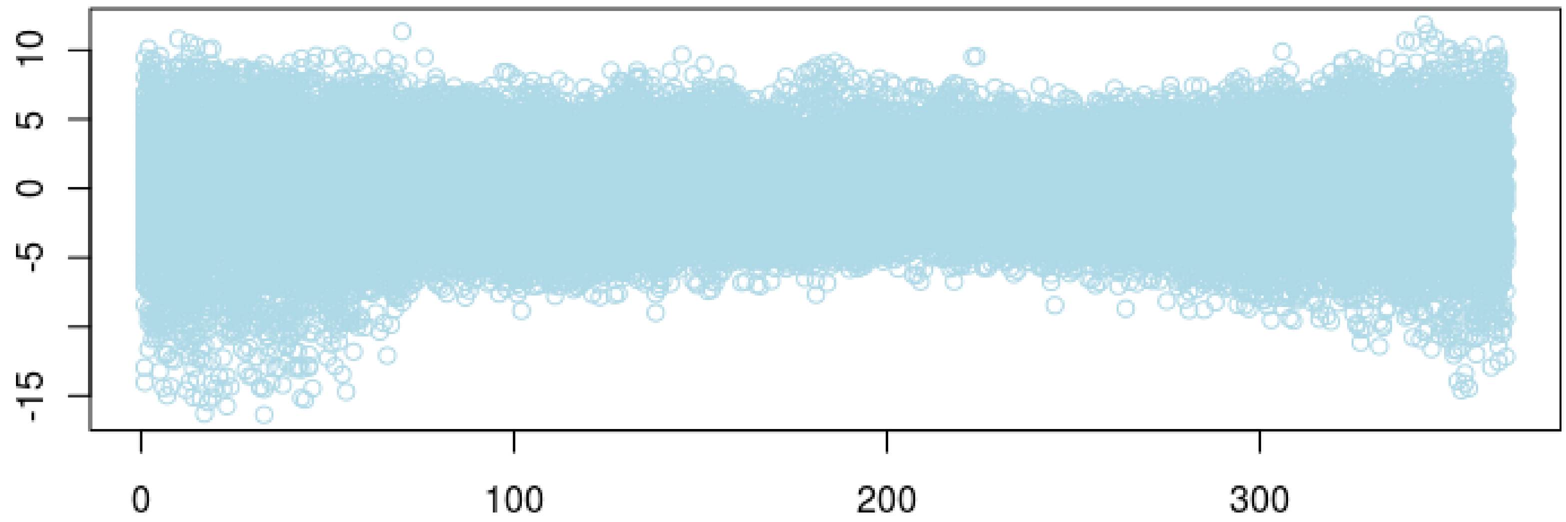
## The residual part (or stationary component)

$\widehat{X}_t$ might look stationary,



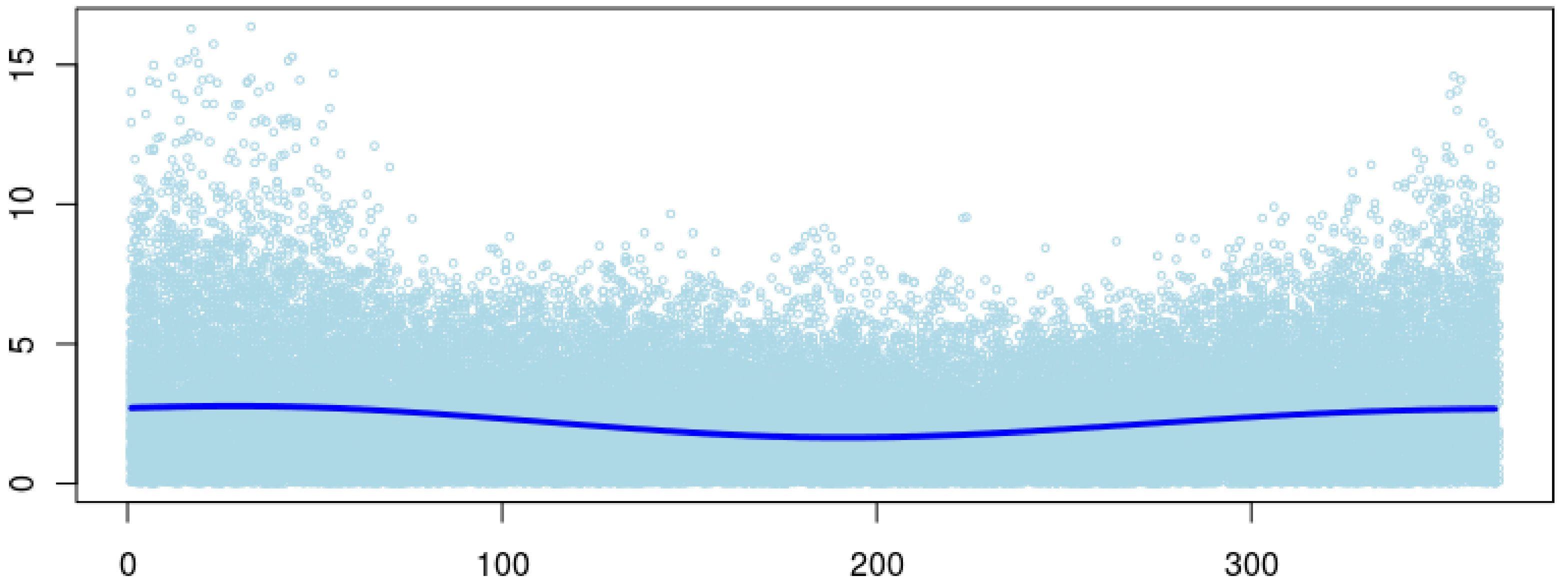
## The residual part (or stationary component)

but the variance of $\widehat{X}_t$ seems to have a seasonal pattern,



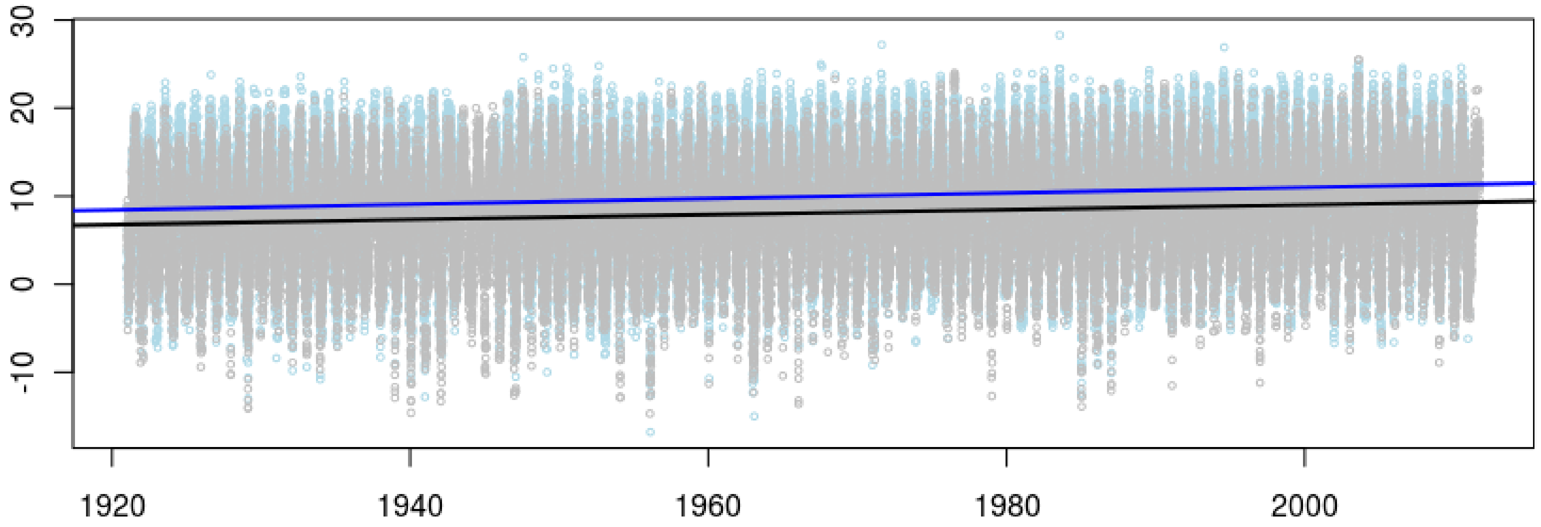
# The residual part (or stationary component)

Dispersion and variance of residuals, graph of $|X_t|$ series



## A spatial model for residuals ?

Consider here two series of temperatures (e.g. Paris and Marseille).



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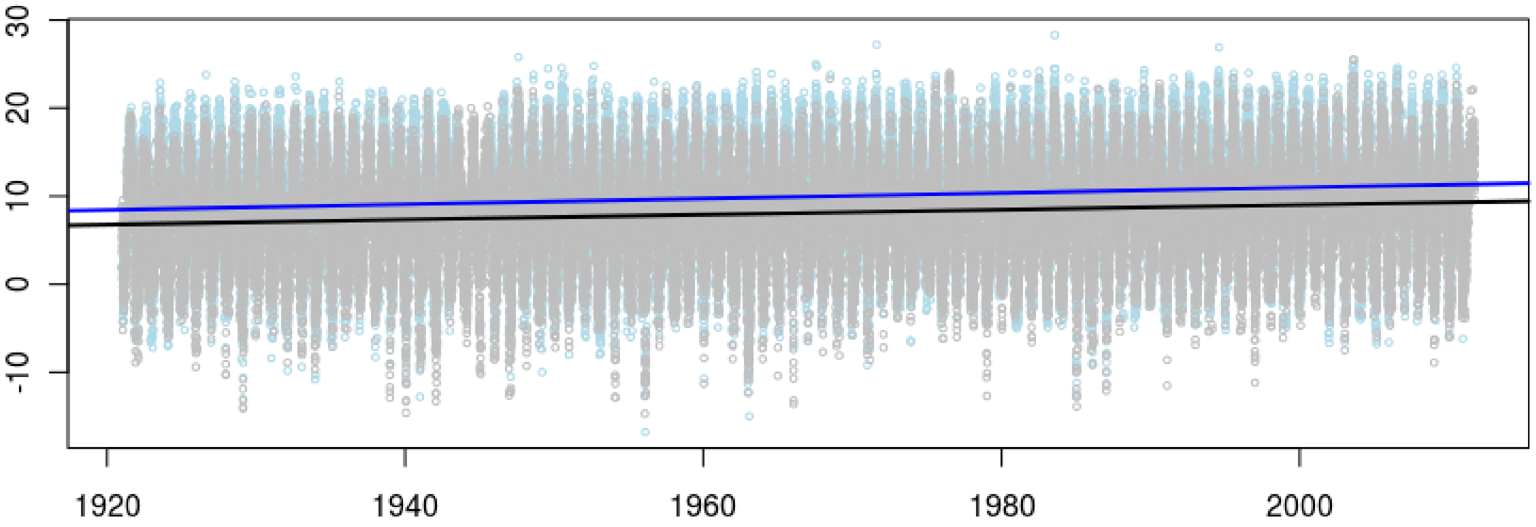
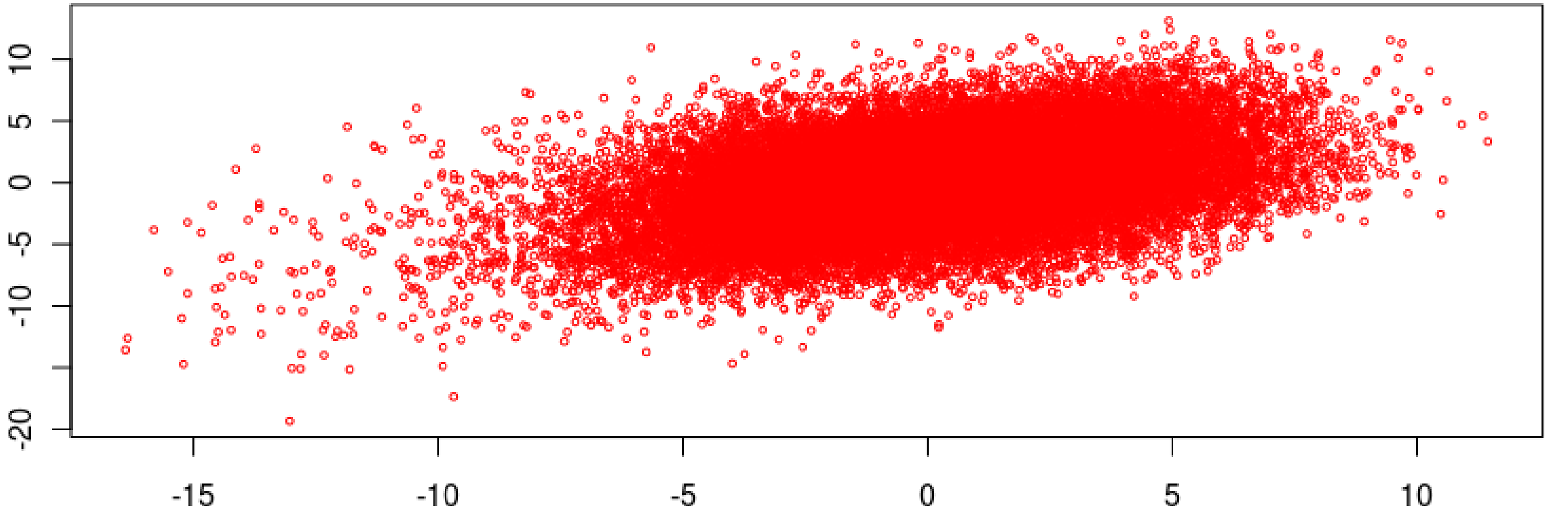


## A spatial model for residuals ?

Consider here two series of temperatures (e.g. Paris and Marseille), and their stationary residuals



## Tail dependence indices

For the lower tail

$$L(z) = \mathbb{P}(U < z, V < z)/z = C(z,z)/z = \mathbb{P}(U < z|V < z) = \mathbb{P}(V < z|U < z),$$

and for the upper tail

$$R(z) = \mathbb{P}(U > z, V > z)/(1-z) = \mathbb{P}(U > z|V > z).$$

see Joe (1990). Define

$$\lambda_U = R(1) = \lim_{z\to 1} R(z) \text{ and } \lambda_L = L(0) = \lim_{z\to 0} L(z).$$

such that

$$\lambda_L = \lim_{u\to 0} \mathbb{P}\left(X \leq F_X^{-1}(u)\,|Y \leq F_Y^{-1}(u)\right),$$

and

$$\lambda_U = \lim_{u\to 1} \mathbb{P}\left(X > F_X^{-1}(u)\,|Y > F_Y^{-1}(u)\right).$$

## Tail dependence indices

Ledford & Tawn (1996) suggested the following alternative approach,

- for independent random variables,
  $\mathbb{P}(X > t, Y > t) = \mathbb{P}(X > t) \times \mathbb{P}(Y > t) = \mathbb{P}(X > t)^2$,
- for comonotonic random variables, $\mathbb{P}(X > t, Y > t) = \mathbb{P}(X > t) = \mathbb{P}(X > t)^1$,

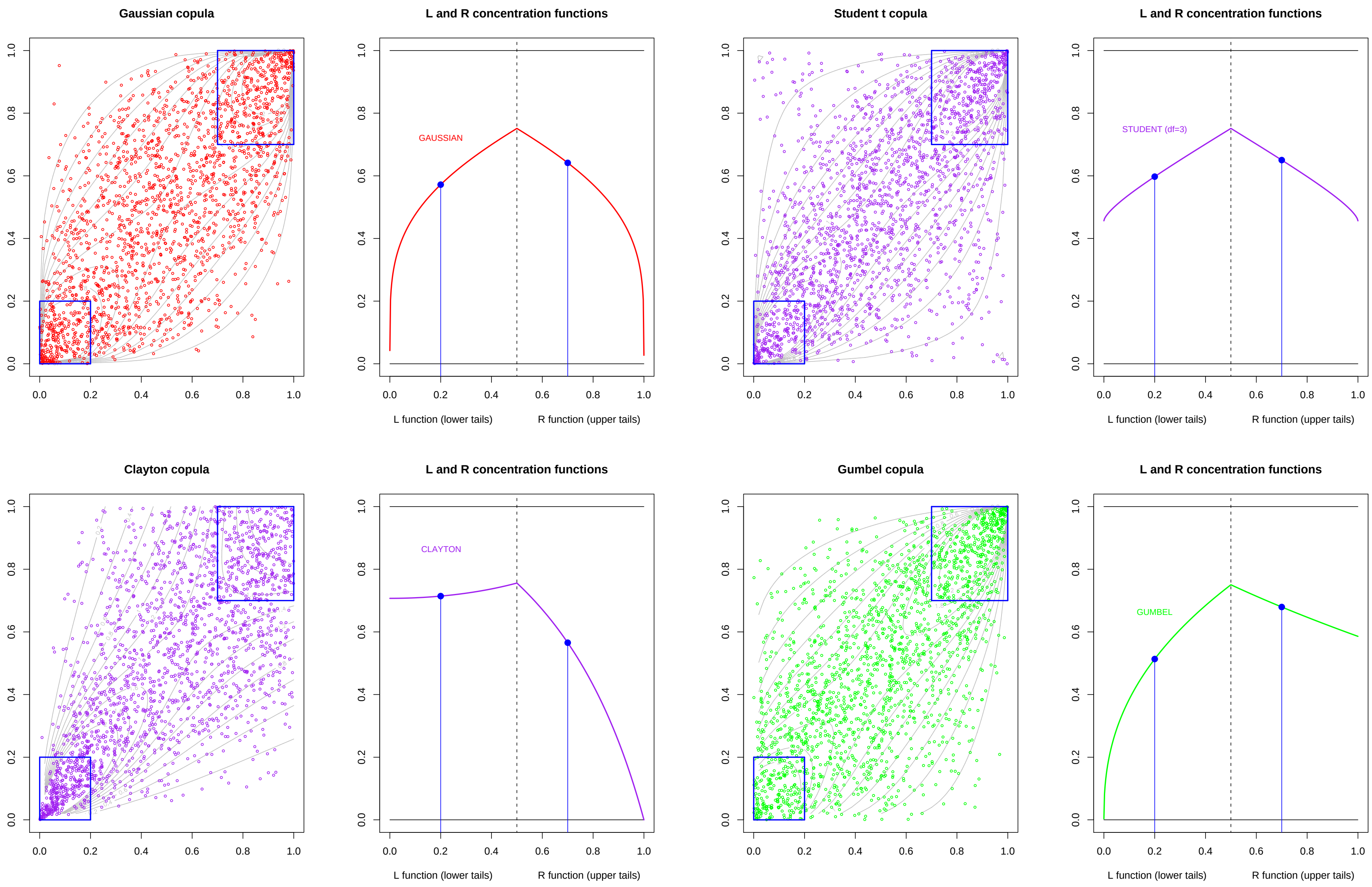
Assume that $\mathbb{P}(X > t, Y > t) \sim \mathbb{P}(X > t)^{1/\eta}$ as $t \to \infty$, where $\eta \in (0, 1]$ will be a tail index.

Define

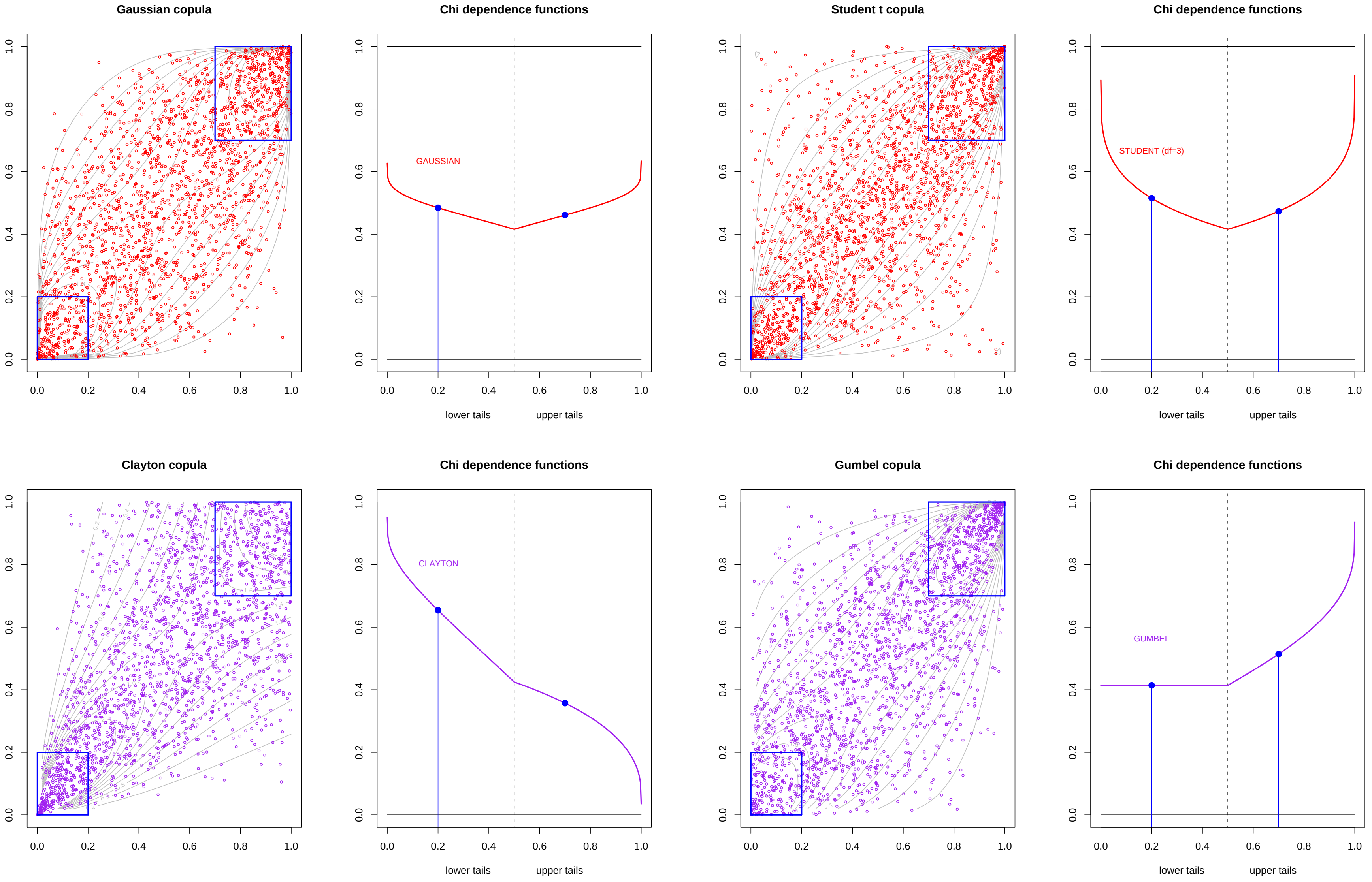
$$\overline{\chi}_U(z) = \frac{2\log(1-z)}{\log C^\star(z,z)} - 1 \text{ and } \overline{\chi}_L(z) = \frac{2\log(1-z)}{\log C(z,z)} - 1$$

with $\eta_U = (1 + \lim_{z\to 0} \overline{\chi}_U(z))/2$ and $\eta_L = (1 + \lim_{z\to 0} \overline{\chi}_L(z))/2$ sont appelés indices de queue supérieure et inférieure, respectivement.

# Tail functions $L(\cdot)$ and $R(\cdot)$



# Tail functions $\overline{\chi}_L(\cdot)$ and $\overline{\chi}_U(\cdot)$



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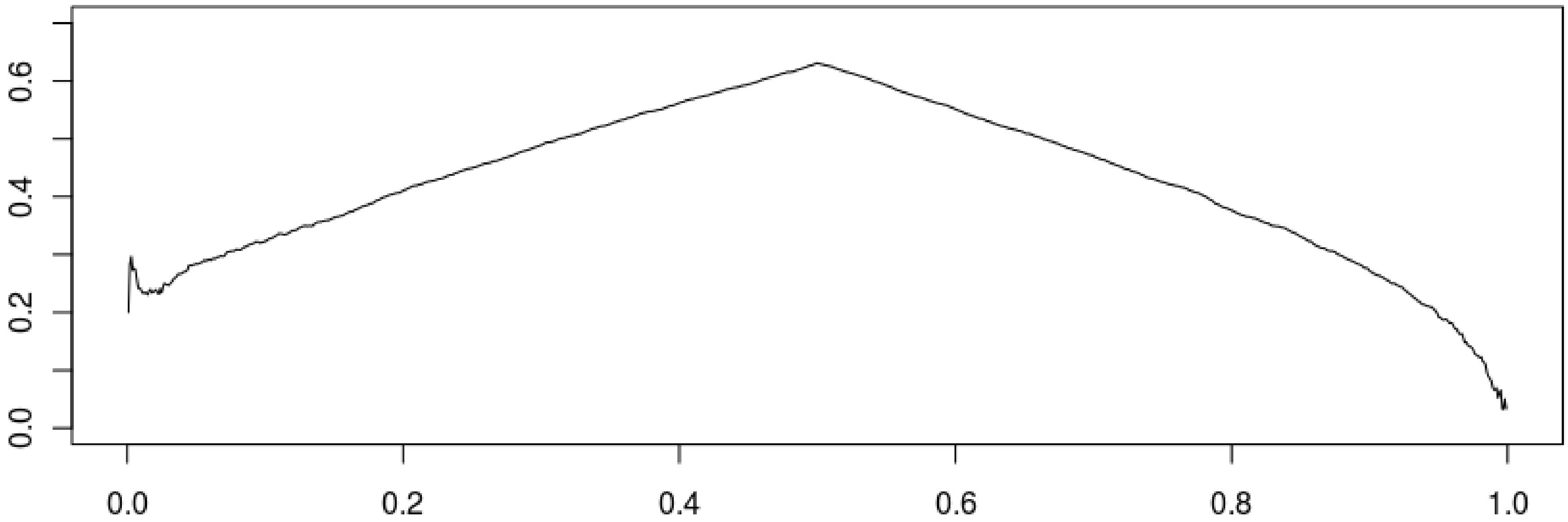
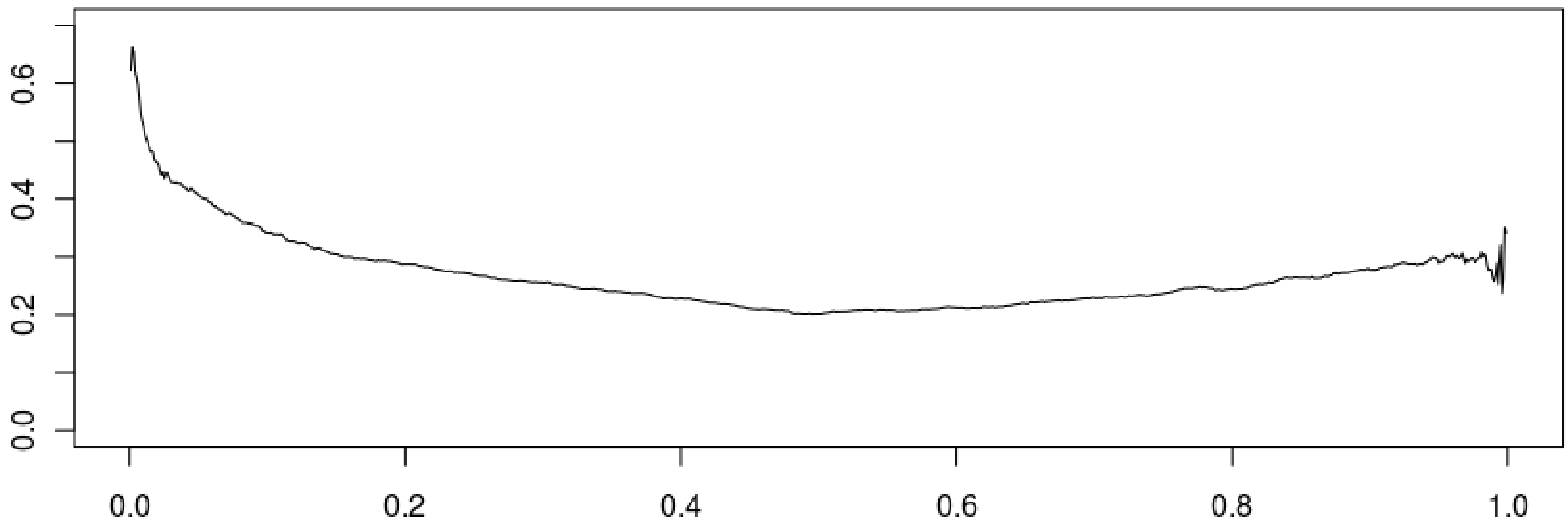


# Tail functions $\overline{\chi}_L(\cdot)$ and $\overline{\chi}_U(\cdot)$

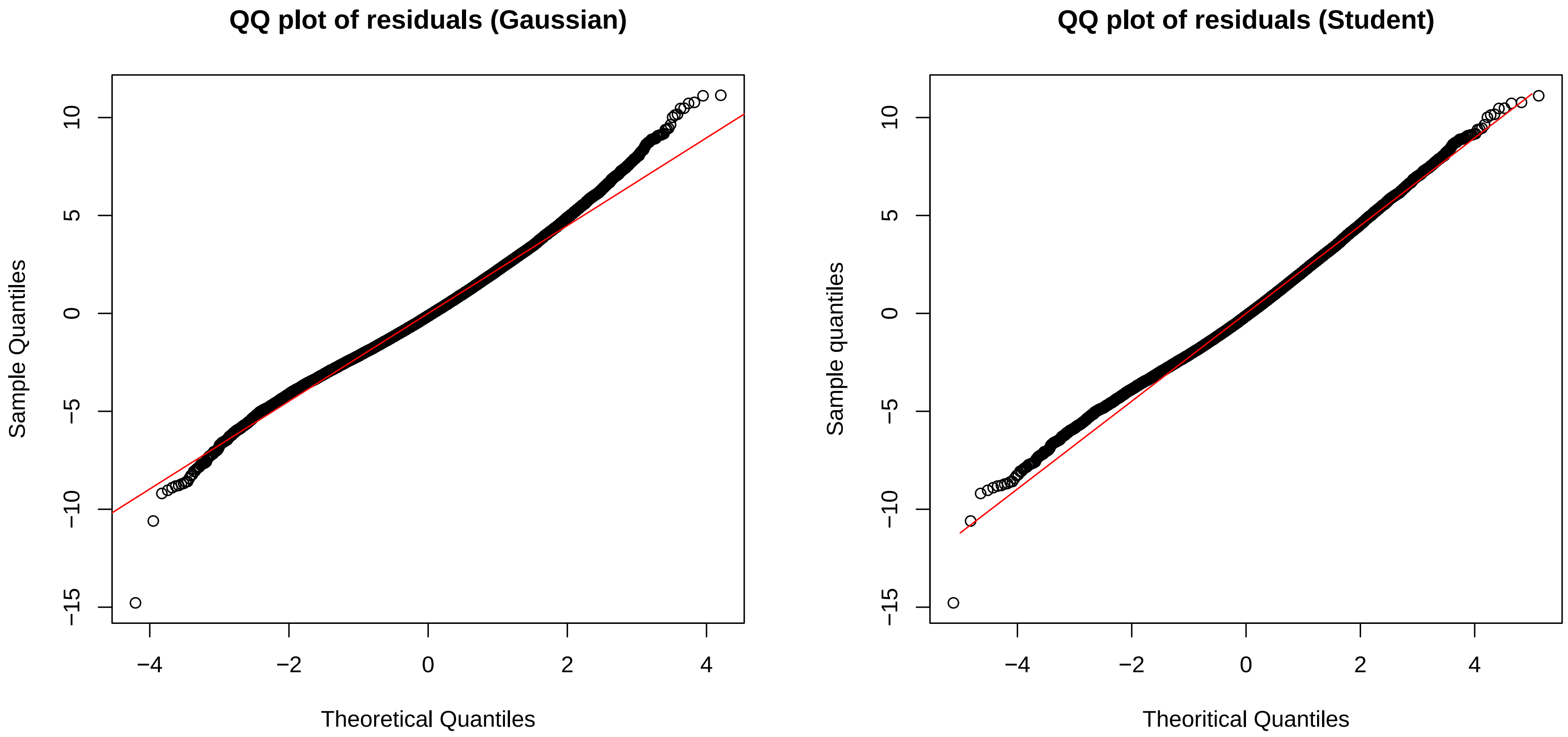


## Dynamics of the residual part

consider an ARMA model to model residuals dynamics

| | $\widehat{\phi}_1$ | $\widehat{\phi}_2$ | $\widehat{\theta}_1$ | $\widehat{\theta}_2$ | $\widehat{\sigma}^2$ |
|---|---|---|---|---|---|
| ARMA(2,2) | 1.4196 (0.0419) | −0.4733 (0.0322) | −0.6581 (0.0419) | −0.1032 (0.00752) | 5.023 |

It is possible to fit a Gaussian or a Student ARMA process,


QQ plot of residuals (Gaussian)
Sample Quantiles
Theoretical Quantiles
−15
−10
−5
0
5
10
−4
−2
0
2
4
QQ plot of residuals (Student)
Sample quantiles
Theoritical Quantiles
−15
−10
−5
0
5
10
−4
−2
0
2
4

## Extremes and discrete data

Recall Fisher-Tippett theorem : let $X_1, \cdots, X_n, \cdots$ be i.i.d. with cdf $F$, with right point $x_F = \sup\{x; F(x) < 1\}$, and define $X_{n:n} = \sup\{X_1, \cdots, X_n\}$. If

$$\lim_{x \to x_F} \frac{1 - F(x)}{1 - F(x^-)} = 1, \tag{1}$$

then there exists 3 possible limits for

$$\lim_{n \to \infty} \mathbb{P}\left(\frac{X_{n:n} - b_n}{a_n} \leq x\right) = G(x),$$

(Fréchet, Gumbel and Weibull), see Leadbetter*et al.*(1983), and we shall say that $F \in \text{MDA}(G)$. But Assumption (1) is *usually* not satisfied for discrete data. Approximations can be derived for standard discrete distributions (Poisson, geometric, etc). Nevertheless, if $X$ is an absolutely continuous random variable, with $x_F = \infty$, with hazard rate satisfying $h(x) = d \log \overline{x} \to 0$ as $x \to \infty$, if $\lfloor X \rfloor$ is a discretized version of $X$, with cdf $\lfloor F \rfloor$, then (1) holds, and further, $\lfloor F \rfloor \in \text{MDA}(G)$ if and only if $F \in \text{MDA}(G)$.

# Generalized Pareto and modeling exceedances

Instead of looking at maximas, use the threshold approch and Pickamds, Balkema, de Haan theorem :

$$\mathbb{P}(X > x + u | X > u) \approx \left(1 + \frac{\xi x}{\beta_u}\right)_+^{-1/\xi}$$

with $x_+ = \max\{x, 0\}$, $\beta_u > 0$ and $\xi$ are respectively scale and shape parameters, in the context of a stationary process $(X_t)$.

For non-stationary processes, Smith (1989) and Davison & Smith (1990) suggested a GPD model with covariate,

$$\mathbb{P}(X > x + u | X > u, \boldsymbol{Z} = \boldsymbol{z}) \approx \left(1 + \frac{\xi(\boldsymbol{z}) x}{\beta_u(\boldsymbol{z})}\right)_+^{-1/\xi(\boldsymbol{z})}$$

See e.g. Chavez-Desmoulins & Davison (2005) for non-parametric (GAM) models.

## Generalized Pareto and modeling exceedances

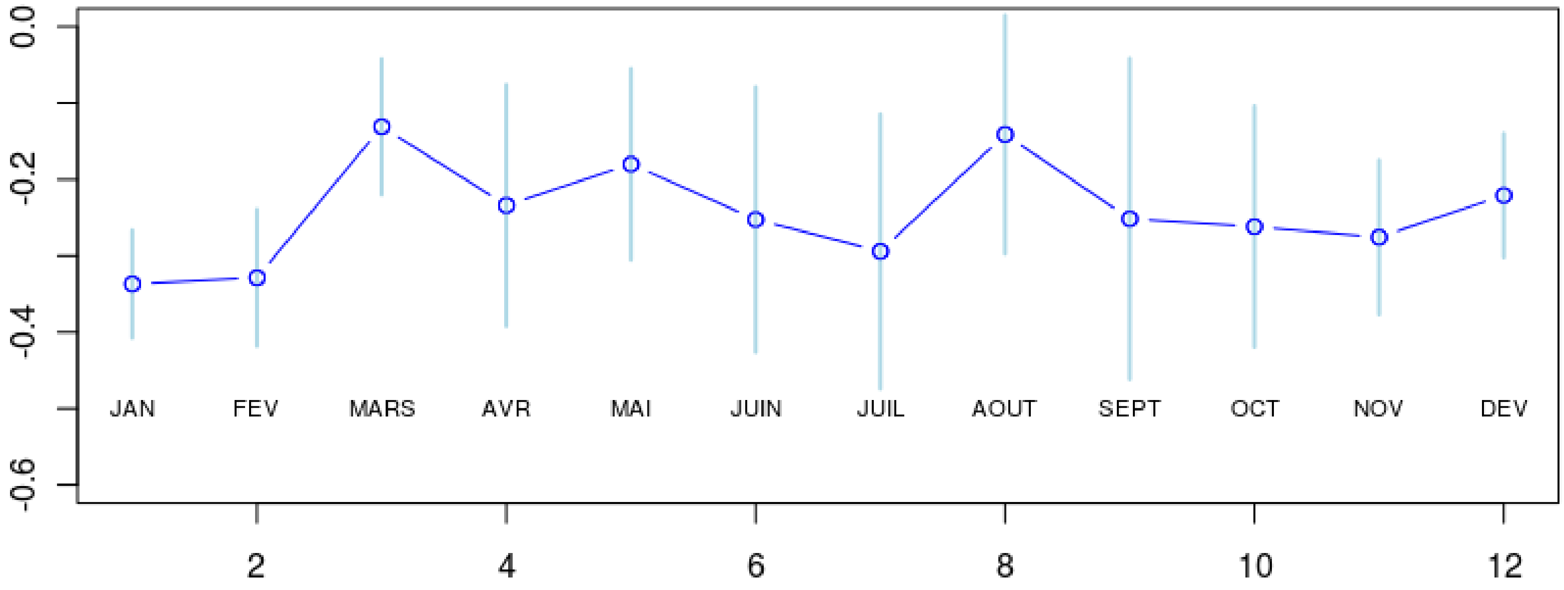
But as point out in Eastoe & Tawn (2008) a strong constraint is that the scale parameter has to satisfy, for any $v > u$,

$$\beta_v(\boldsymbol{z}) = \beta_u(\boldsymbol{z}) + (v - u)\xi(\boldsymbol{z}).$$

Coelho *et al.* (2008) and Kyslý*et al.* (2010) suggested to use a time varying threshold $u(\boldsymbol{z})$, while Eastoe & Tawn (2008) suggested preprocessing techniques, and to assume that $X_t = \mu_t + \sigma_t \tilde{X}_t$, with $\tilde{X}_t$ stationary.

## Generalized Pareto and modeling exceedances

What about tails of $|X_t|$ ?



## Generalized Pareto and modeling exceedances

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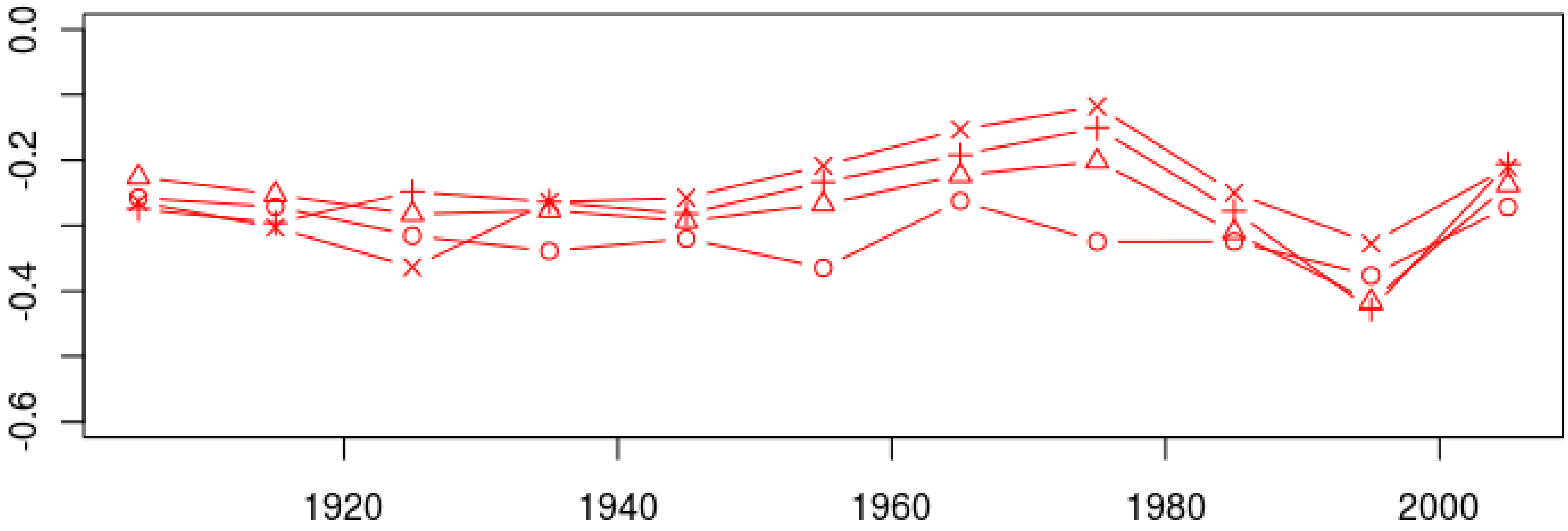



different thresholds $u$.

## Long range dependence ?

Smith (1993) or Dempster & Liu (1995) suggested that, on a long period, the average annual temperature should be decomposed as follows

- an increasing *linear* trend,
- a random component, with *long range dependence*.

E.g.$(Y_t)$ such that $\Phi(L)(1-L)^d Y_t = \Theta(L)\varepsilon_t$ where $d \in (-1/2, 1/2)$, and where

$$(1-L)^d = 1 + \sum_{j=1}^{\infty} \frac{d(d-1)\cdots(d-j+1)}{j!}(1-)^j L^j.$$

i.e. ARFIMA$(p,d,q)$, *fractional* processes (see e.g. Hurst (1951) or Mandelbrot (1965))

E.g.$(Y_t)$ such that

$$\Phi(L)(1-2uL+L^2)^d Y_t = \Theta(L)\varepsilon_t,$$

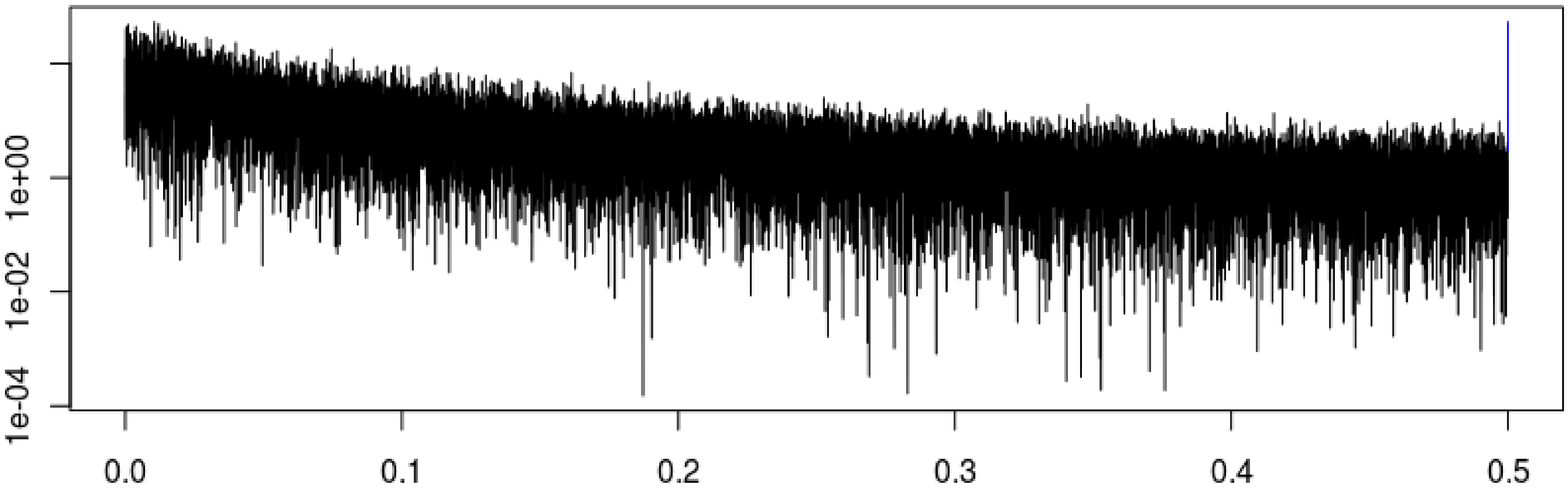
i.e. GARMA$(p,d,q)$ from Hosking (1981) Gegenbauer's frequency, defined as $\omega = \cos^{-1}(u)$, is closely related to the seasonality of the series. Here, $\widehat{u} = 2\pi/365$ (because of the annual cycle of temperature).

A stationary process $(Y_t)_{t\in\mathbb{Z}}$ is said to have long range dependence if

$$\sum_{h=1}^{\infty}|\rho_X(h)|=\infty,$$

and short range dependence if not.

Smith (1993) "*we do not believe that the autoregressive model provides an acceptable method for assessing theses uncertainties*" (on temperature series)



# Long range dependence and regression

Gray, Zhang & Woodward (1989) proposed an extension to model persistent *seasonal* series, using Gegenbauer's polynomial $G_n^d(\cdot)$, defined as

$$G_n^d(x) = \frac{(-2)^n}{n!}\frac{\Gamma(n+d)\Gamma(n+2d)}{\Gamma(d)\Gamma(2n+2d)}(1-x^2)^{-\alpha+1/2}\frac{d^n}{dx^n}\left[(1-x^2)^{n+d-1/2}\right].$$

and such that $G_n^d(\cdot)$ is the unique polynomial of degree $n$ such that

$$(1-2uZ+Z^2)^d = \sum_{n=0}^{\infty} G_n^d(u)\cdot Z^n.$$

If $d \in (0,1/2)$, and $|u|<1$ then

$$\rho(h) \sim h^{2d-1}\sin([\pi - \arccos(u)]h) \text{ as } h \to \infty.$$

# Long range dependence and regression

If $\gamma_X(\cdot)$ denotes the autocovariance function of a stationary process $(X_t)_{t\in\mathbb{Z}}$ ,

$$Var(\overline{X}_n) = \frac{\gamma_X(0)}{n} + \frac{2}{n}\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)\gamma(k),$$

where $\overline{X}_n$ is the standard empirical mean of a sample $\{X_1, ..., X_n\}$ (see Brockwell & Davis (1991), or Smith (1993)).

Furthermore, if autocovariance function satisfies $\gamma(h) \sim a \cdot h^{2d-1}$ as $h \to \infty$, then

$$Var(\overline{X}_n) \sim \frac{a}{d(2d-1)} \cdot n^{2d-2},$$

as derived in Samarov & Taqqu (1988).

And further, the ordinary least squares estimator of the slope $\beta$ (in the case where the $X_i$'s are regressed on some covariate $Y$) is still

$$\widehat{\beta} = \frac{\sum X_i(Y_i - \overline{Y}_n)}{\sum(Y_i - \overline{Y}_n)^2}.$$

As shown in Yajima (1988), and more generally in Yajima (1991) in the case of general regressors,

$$Var(\widehat{\beta}) \sim \frac{36a(1-d)}{d(1+d)(2d+1)} \cdot n^{2d-4}.$$

## Long range dependence and extremes

From Leadbetter (1974), let $X_1, \cdots, X_n, \cdots$ be a stationnary process, and denote $X_{n:n} = \max\{X_1, \cdots, X_n\}$ ; if there exists sequences $(a_n)$ and $(b_n)$ such that

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{X_{n:n} - b_n}{a_n} \leq x\right) = G(x),$$

and if so called $D(a_n z + b_n)$ holds for all $z \in \mathbb{R}$ (i.e. not long range dependence), then $G$ is a GEV distribution.

**Remark** but parameters are not necessarily the same as an i.i.d. sequence with the same margins (see extremal index concept.

Condition $D(u_n)$ from Leadbetter *et al.* (1983) : for all $p, q$ and $n$, and indices

$$1 \leq \underbrace{i_1 < i_2 < \cdots < i_p}_{I} < \underbrace{j_1 < j_2 < \cdots < j_q}_{J} \leq n,$$

such that $j_1 - i_p \geq m$,

$$\left|\mathbb{P}\left(\max_{k\in I\cup J}\{X_k\} \leq u_n\right) - \mathbb{P}\left(\max_{i\in I}\{X_i\} \leq u_n\right) \cdot \mathbb{P}\left(\max_{j\in J}\{X_j\} \leq u_n\right)\right| \leq \alpha(n,m),$$

where $\alpha(n,m) \to 0$ as $n \to \infty$, for some $m = o(n)$.

**Remark** Note that

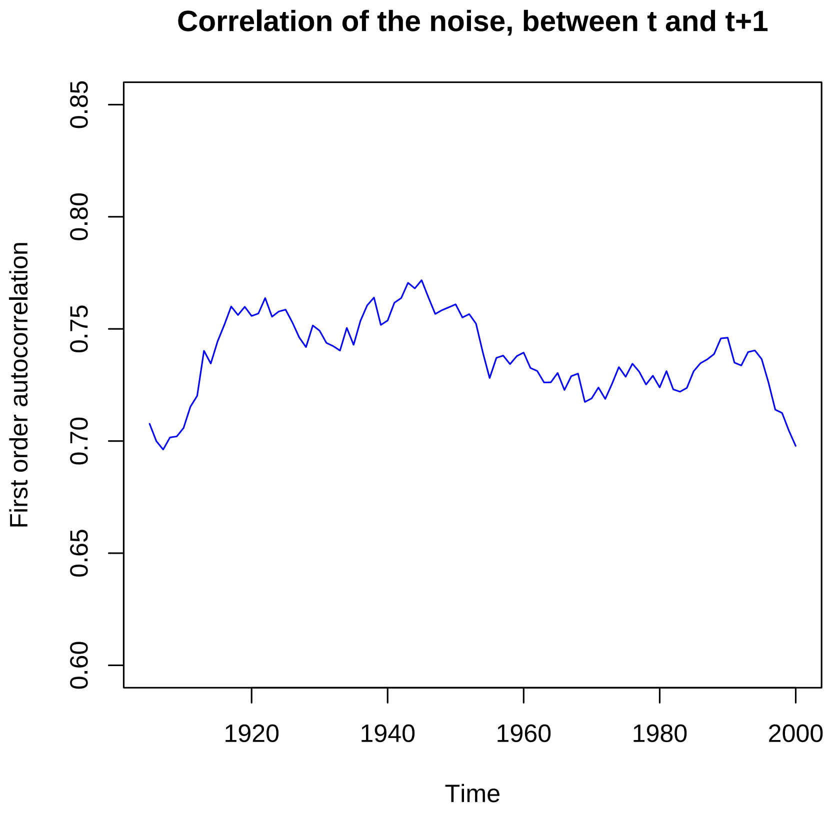
$$\left|\mathbb{P}\left(\max_{k\in I\cup J}\{X_k\} \leq u_n\right) - \mathbb{P}\left(\max_{i\in I}\{X_i\} \leq u_n\right) \cdot \mathbb{P}\left(\max_{j\in J}\{X_j\} \leq u_n\right)\right| = 0$$

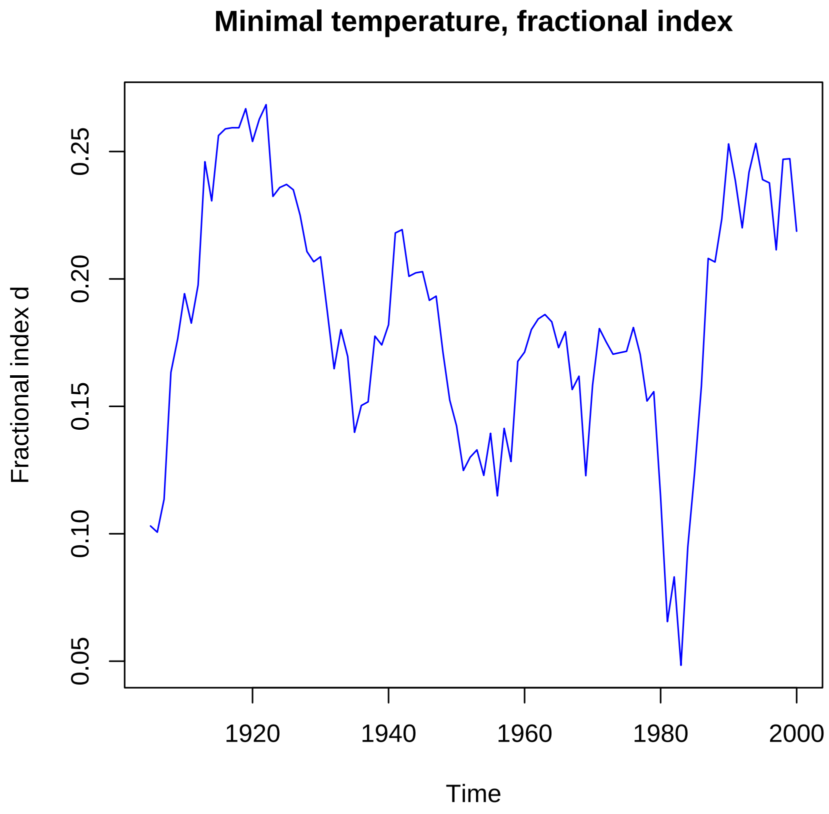
means independence.

**Remark** For a Gaussian sequence, with autocorrelation function $\rho(h)$, $D(u_n)$ is satisfied if $\rho(n) \cdot \log(n) \to 0$ as $n \to \infty$. I.e. weaker than geometric decay for $AR$ processes.

# Stability of the dynamics

First order autocorrelation of the noise ($\rho(1) = \text{corr}(X_t, X_{t-1})$) and estimation of the fractional index $d$.



## On return periods

Depends on scenarios for the increasing trend,

- an optimistic scenario, where we assume that there will be no more increasing trend in the future,
- a pessimistic scenario, where we assume that the trend will remain, with the same slope.

Depends on the definition of the heat wave

- during 11 consecutive days, the temperature was higher than $19°$ C (type (A)),
- during 3 consecutive days, the temperature was higher than $24°$ C (type (B)).

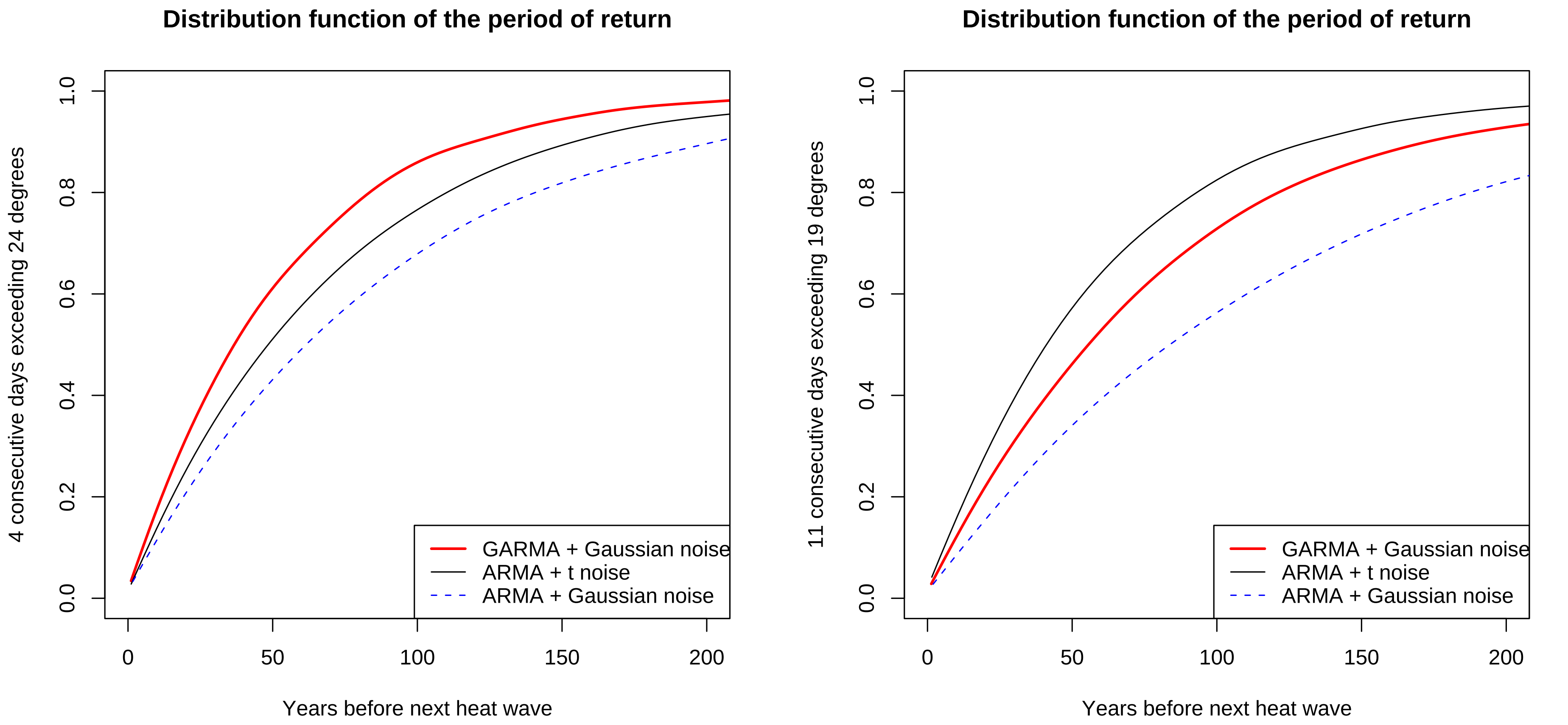
| | short memory short tail noise | short memory heavy tail noise | long memory short tail noise |
|---|---|---|---|
| optimistic | 88 years | 69 years | 53 years |
| pessimistic | 79 years | 54 years | 37 years |

Table 1 – Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (A)).

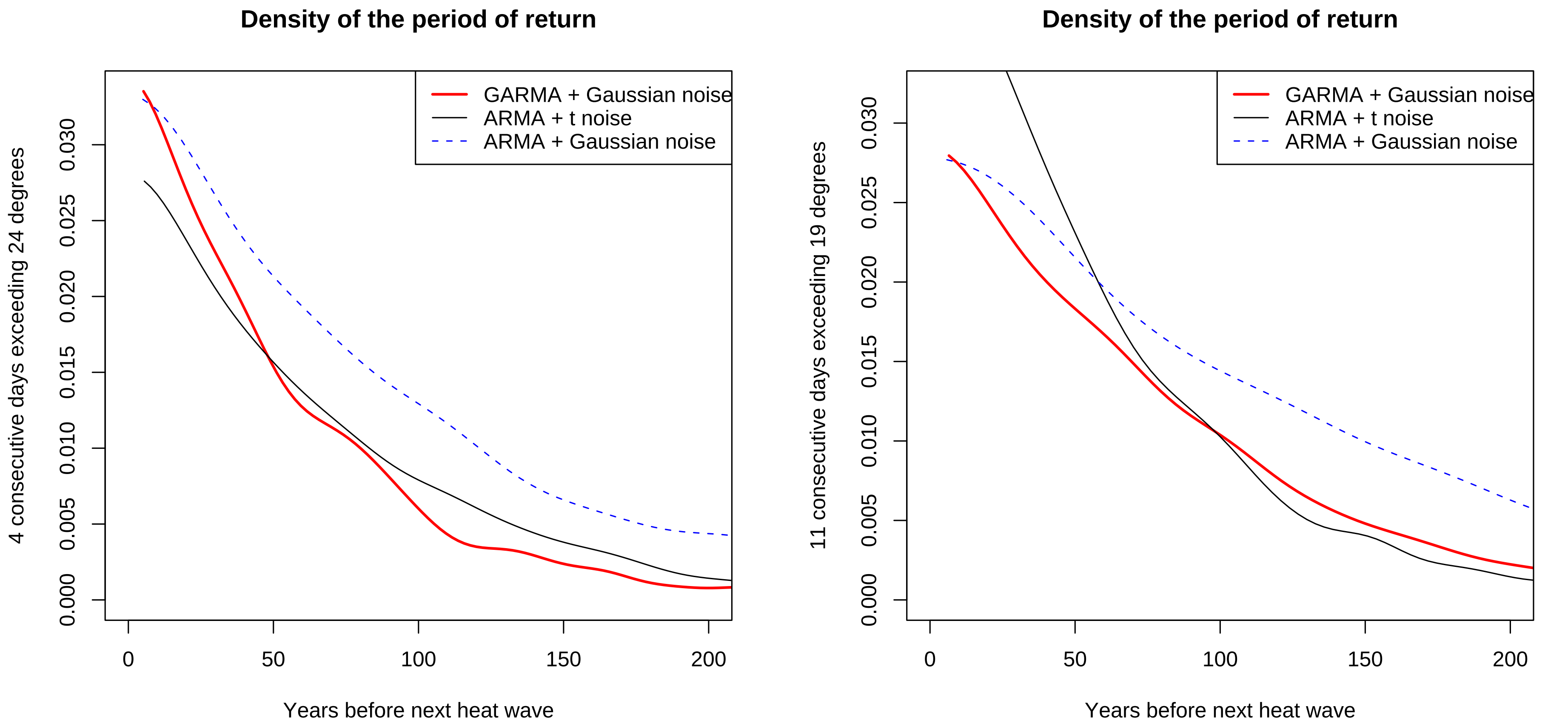
| | short memory<br>short tail noise | short memory<br>heavy tail noise | long memory<br>short tail noise |
|---|---|---|---|
| optimistic | 115 years | 59 years | 76 years |
| pessimistic | 102 years | 51 years | 64 years |

Table 2 – Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (B)).

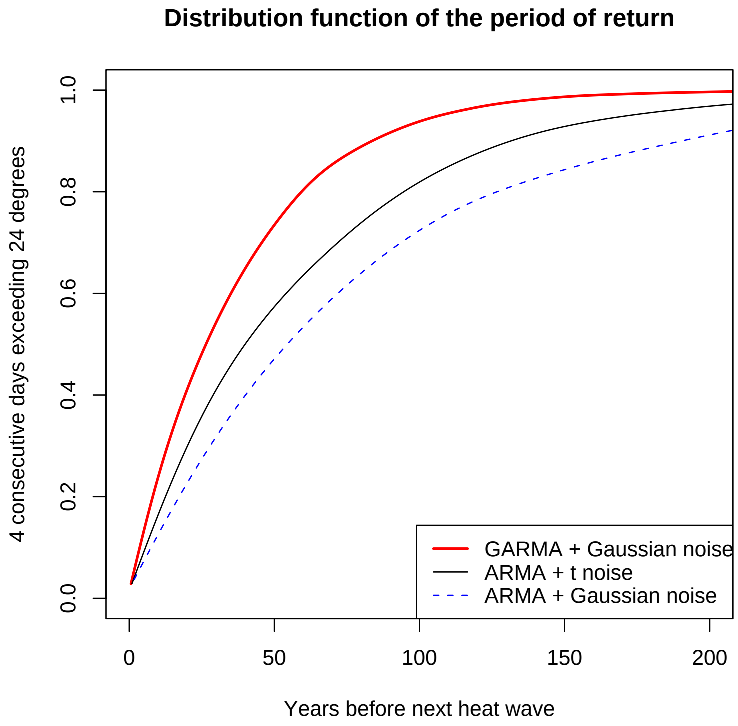
# On return periods, optimistic scenario

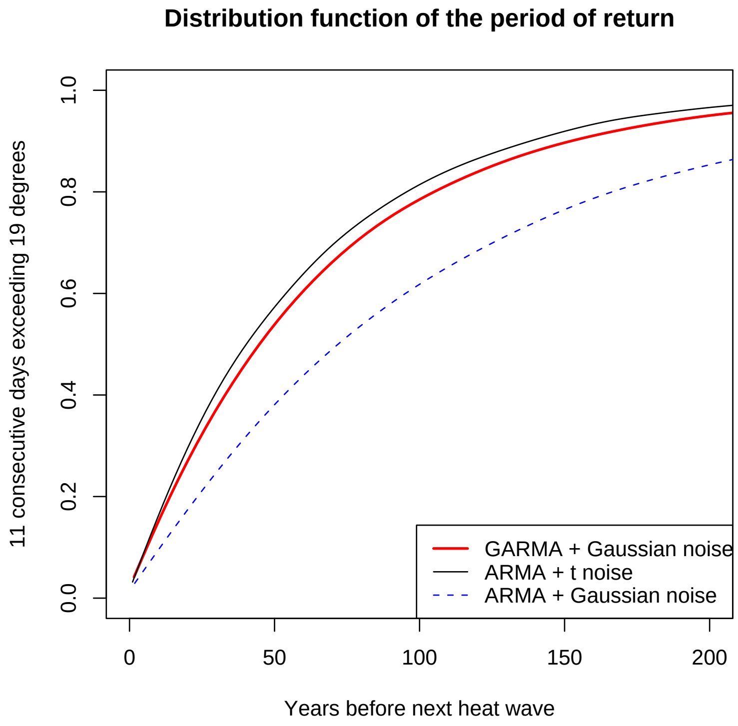


# On return periods, optimistic scenario

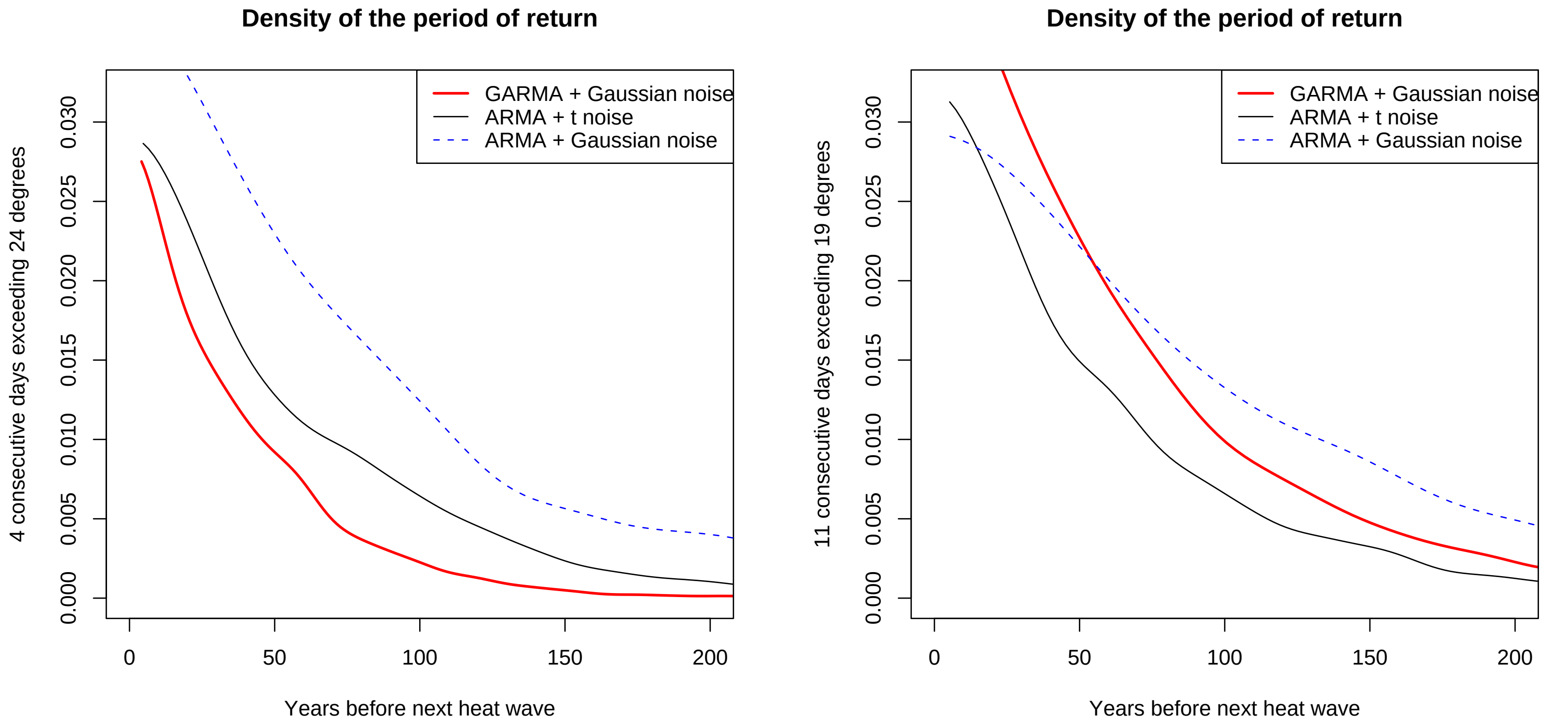


# On return periods, optimistic scenario



# On return periods, optimistic scenario



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