Actuarial Pricing Game

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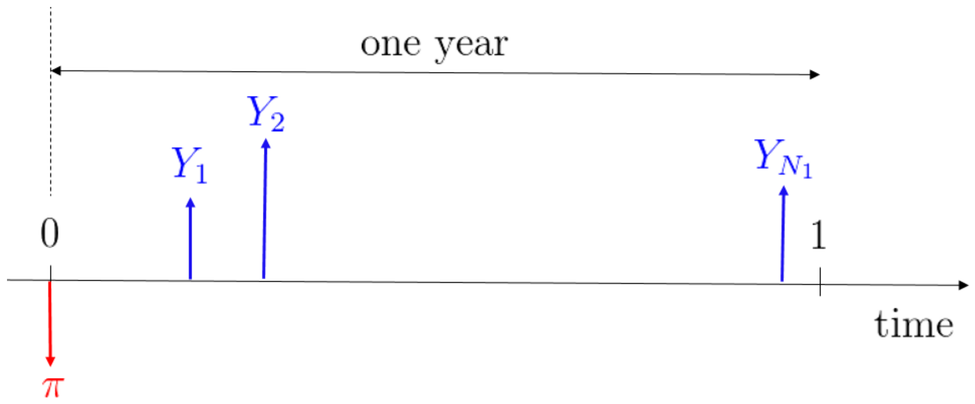
Dependence Modelling with Applications in Finance and Insurance Conference

Insurance Ratemaking *"the contribution of the many to the misfortune of the few"*



Finance: risk neutral valuation $\pi = \mathbb{E}_{\mathbb{Q}}\big[S_1\big|\mathcal{F}_0\big] = \mathbb{E}_{\mathbb{Q}_0}\big[S_1\big]$, where $S_1 = \displaystyle\sum_{i=1}^{N_1} Y_i$

Insurance: risk sharing (pooling) $\pi = \mathbb{E}_{\mathbb{P}}\big[S_1\big]$

or, with segmentation $\pi(\omega) = \mathbb{E}_{\mathbb{P}}\big[S_1\big|\Omega = \omega\big]$ for some (unobservable?) risk factor Ω

imperfect information given some (observable) risk variables $\boldsymbol{X} = (X_1, \cdots, X_k)$
$\pi(\boldsymbol{x}) = \mathbb{E}_{\mathbb{P}}\big[S_1\big|\boldsymbol{X} = \boldsymbol{x}\big]$

Insurance Ratemaking Competition

In a competitive market, insurers can use different sets of variables and different models, with GLMs, $N_t|\boldsymbol{X} \sim \mathcal{P}(\lambda_{\boldsymbol{X}} \cdot t)$ and $Y|\boldsymbol{X} \sim \mathcal{G}(\mu_{\boldsymbol{X}}, \varphi)$

$$z_j = \widehat{\pi}_j(\boldsymbol{x}) = \widehat{\mathbb{E}}\big[N_1\big|\boldsymbol{X} = \boldsymbol{x}\big] \cdot \widehat{\mathbb{E}}\big[Y\big|\boldsymbol{X} = \boldsymbol{x}\big] = \underbrace{\exp(\widehat{\boldsymbol{\alpha}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp(\widehat{\boldsymbol{\beta}}^{\mathsf{T}}\boldsymbol{x})}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{X}}, \varphi)}$$

(see Kaas *et al.* (2008)) or any other statistical model (see Hastie *et al.* (2009))

$$z_j = \widehat{\pi}_j(\boldsymbol{x}) \text{ where } \widehat{\pi}_j \in \underset{m \in \mathcal{F}_j : \Pi\mathcal{X}_j \to \mathbb{R}}{\text{argmin}} \left\{ \sum_{i=1}^{n} \ell(s_i, m(\boldsymbol{x}_i)) \right\}$$

With d competitors, each insured i has to choose among d premiums, $\boldsymbol{\pi}_i = \big(\widehat{\pi}_1(\boldsymbol{x}_i), \cdots, \widehat{\pi}_d(\boldsymbol{x}_i)\big) \in \mathbb{R}_+^d$.

More and more price differentiation ?

Consider $\pi_1 = \mathbb{E}[S_1]$ and $\pi_2(\boldsymbol{x}) = \mathbb{E}[S_1|\boldsymbol{X} = \boldsymbol{x}]$

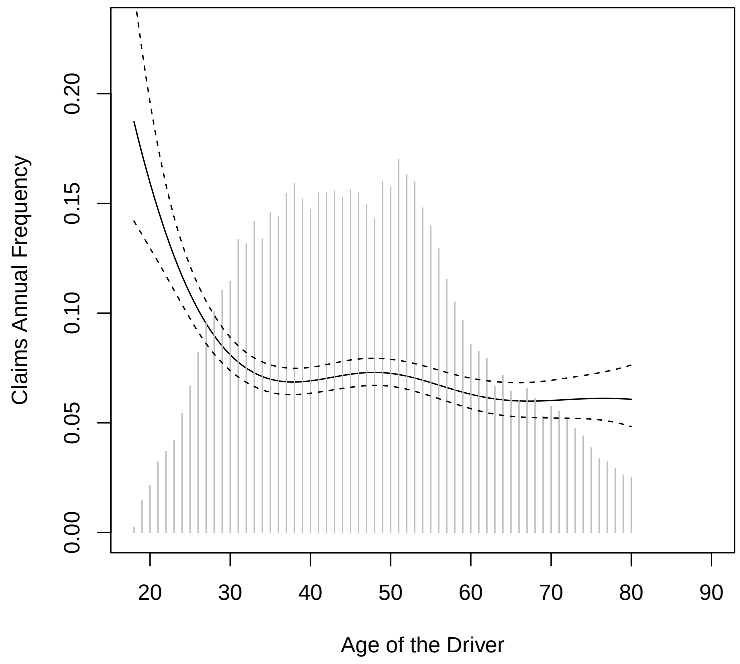
Observe that $\mathbb{E}[\pi(\boldsymbol{X})] = \sum_{\boldsymbol{x}\in\mathcal{X}} \pi(\boldsymbol{x}) \cdot \mathbb{P}[\boldsymbol{x}] = \pi_1$

$$= \sum_{\boldsymbol{x}\in\mathcal{X}_1} \pi(\boldsymbol{x}) \cdot \mathbb{P}[\boldsymbol{x}] + \sum_{\boldsymbol{x}\in\mathcal{X}_2} \pi(\boldsymbol{x}) \cdot \mathbb{P}[\boldsymbol{x}]$$

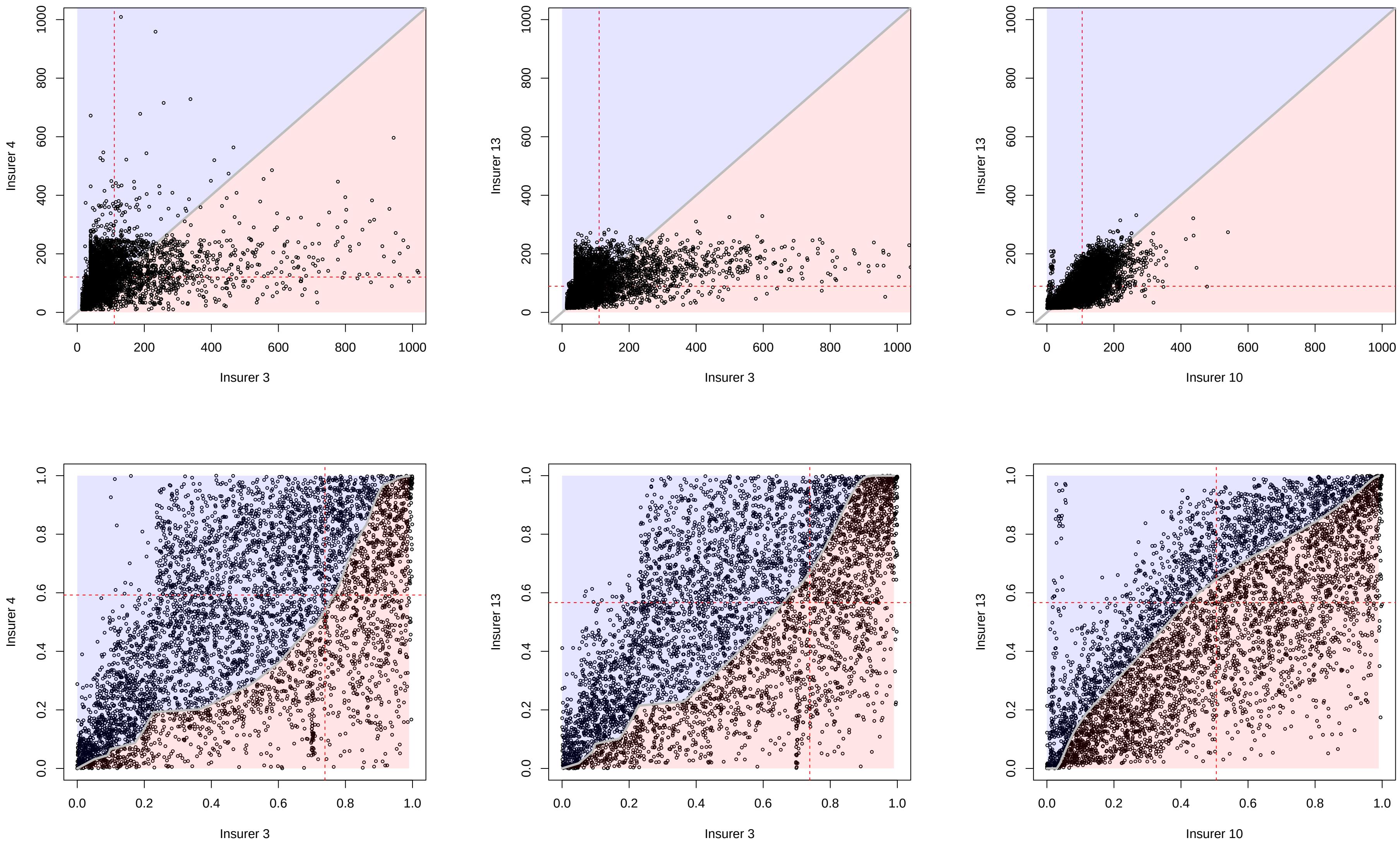
- Insured with $\boldsymbol{x} \in \mathcal{X}_1$: choose Ins1
- Insured with $\boldsymbol{x} \in \mathcal{X}_2$: choose Ins2

$$\sum_{\boldsymbol{x}\in\mathcal{X}_1} \pi_1(\boldsymbol{x}) \cdot \mathbb{P}_{\text{Ins1}}[\boldsymbol{x}] \neq \mathbb{E}[S|\boldsymbol{X} \in \mathcal{X}_1] = \mathbb{E}_{\text{Ins1}}[S]$$

$$\sum_{\boldsymbol{x}\in\mathcal{X}_2} \pi_2(\boldsymbol{x}) \cdot \mathbb{P}_{\text{Ins2}}[\boldsymbol{x}] = \mathbb{E}[S|\boldsymbol{X} \in \mathcal{X}_2] = \mathbb{E}_{\text{Ins2}}[S]$$



Insurance Ratemaking Competition (episode 1, season 3) **comonotonicity?**



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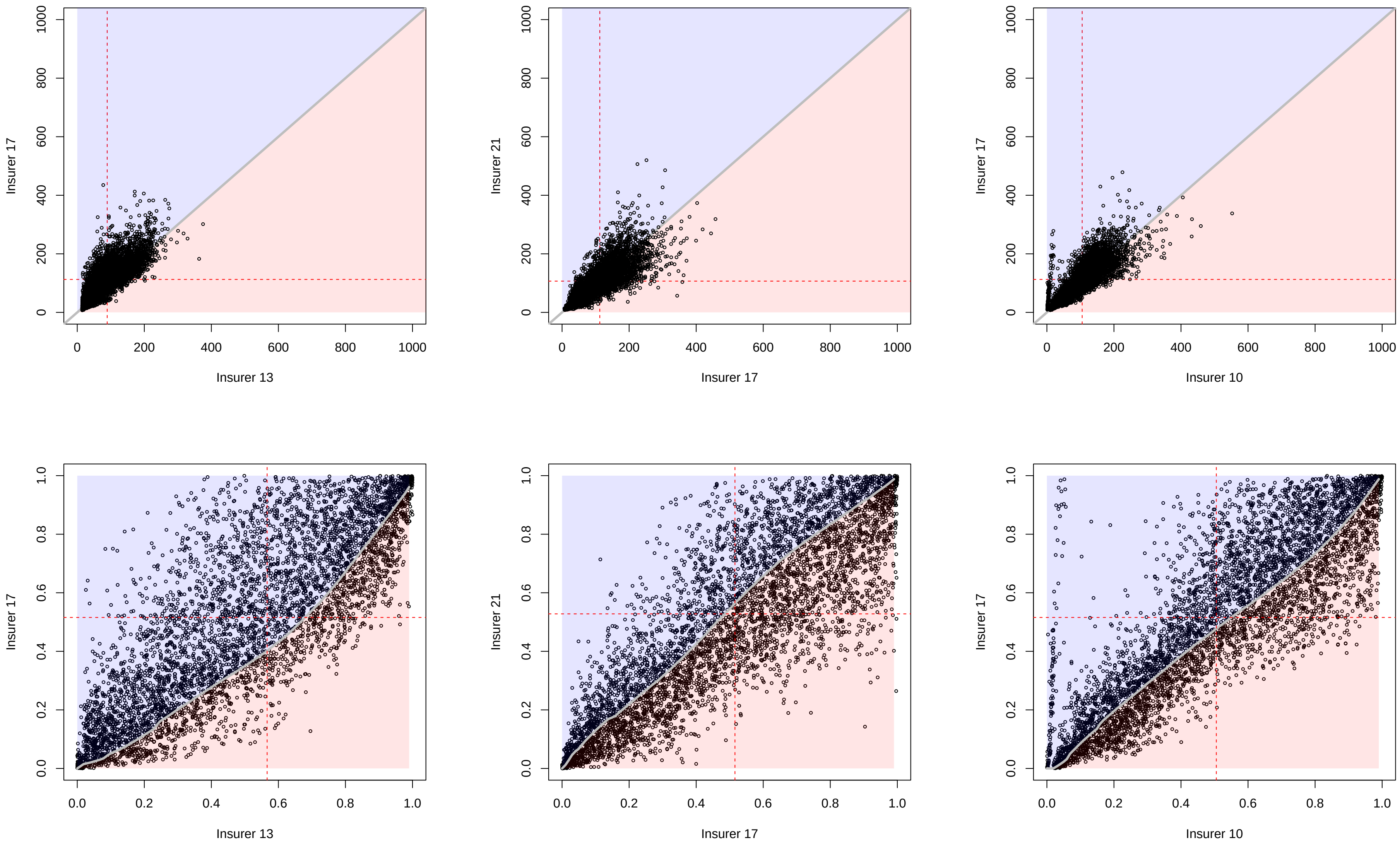

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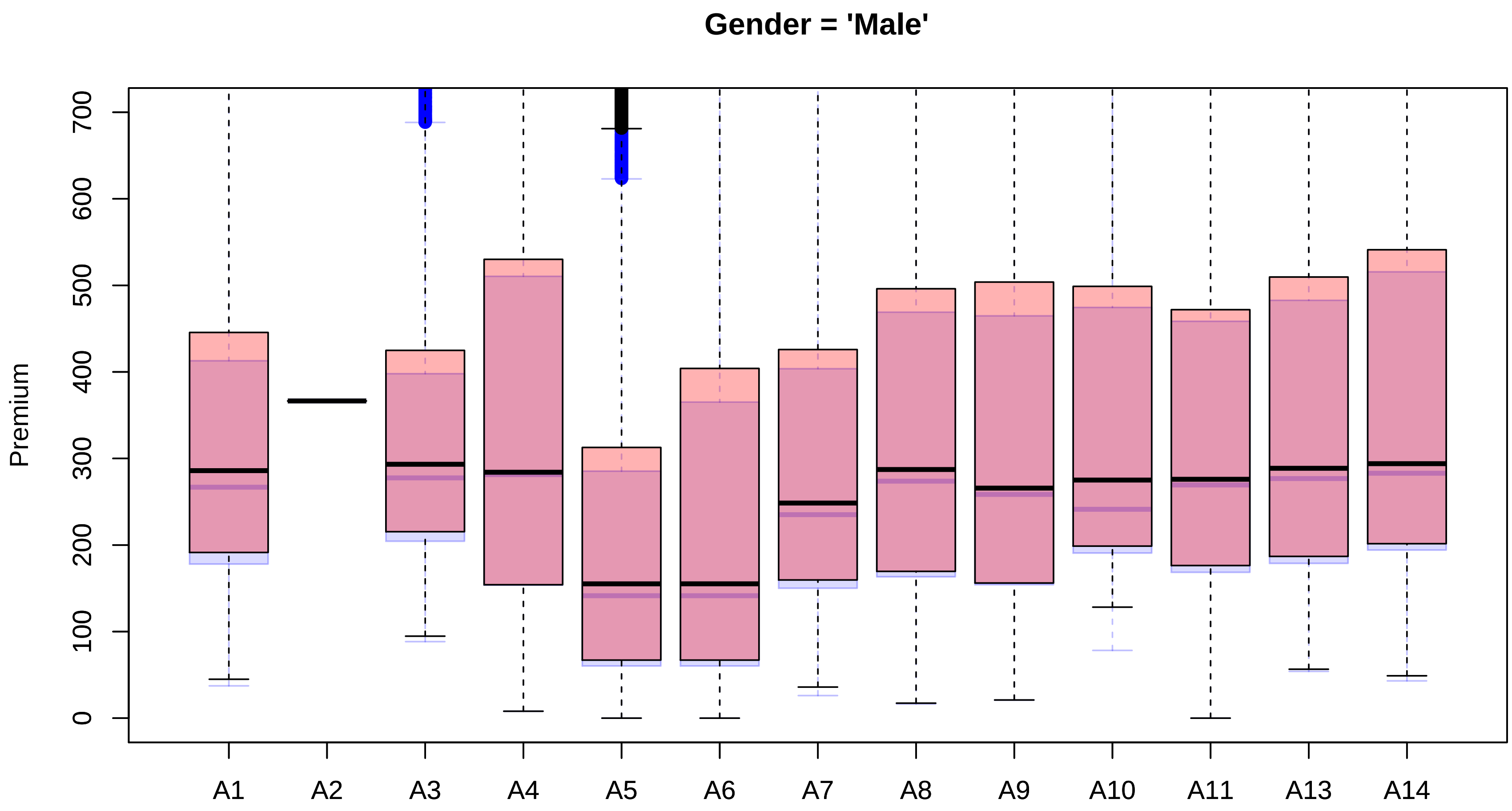

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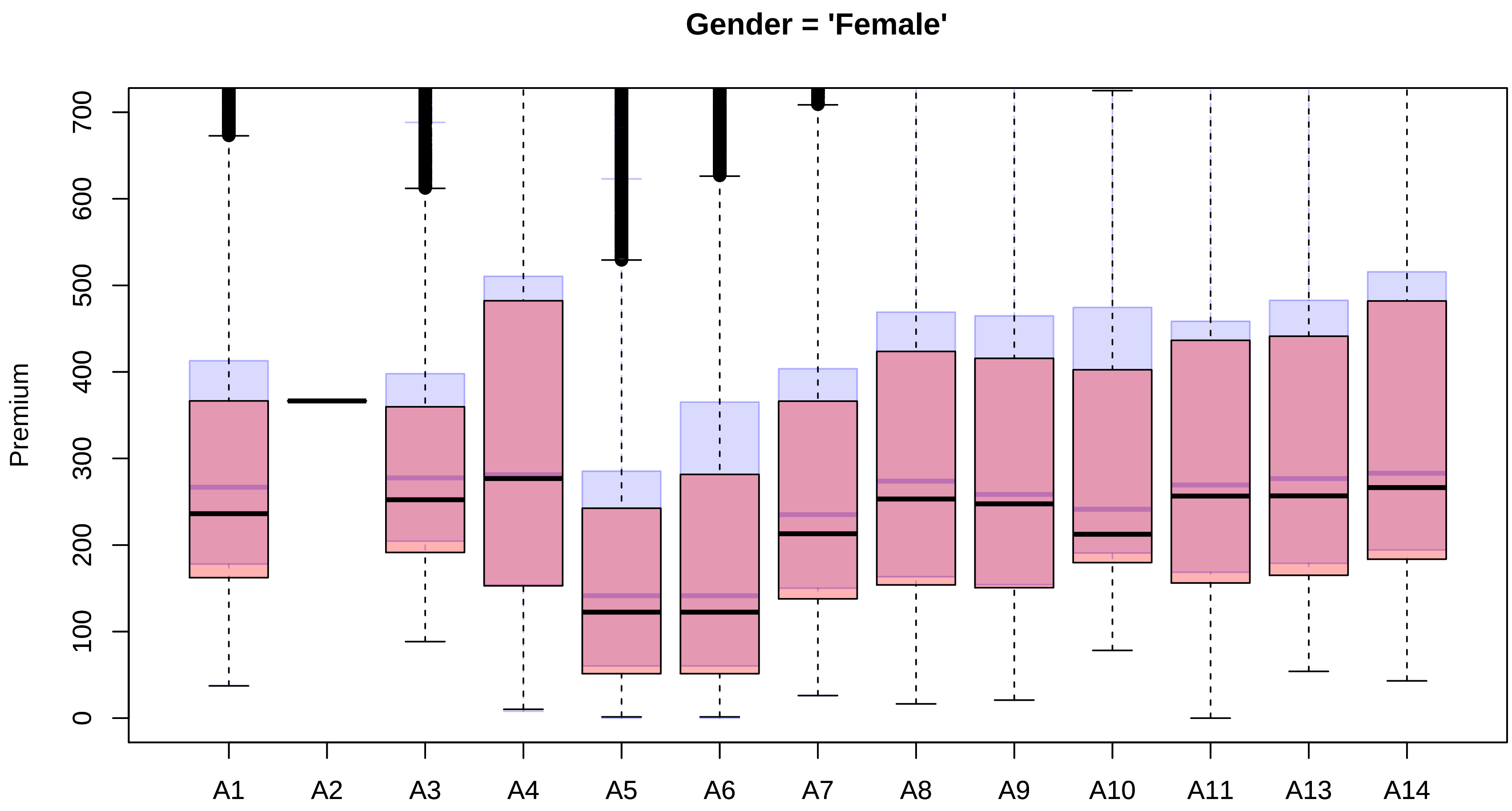

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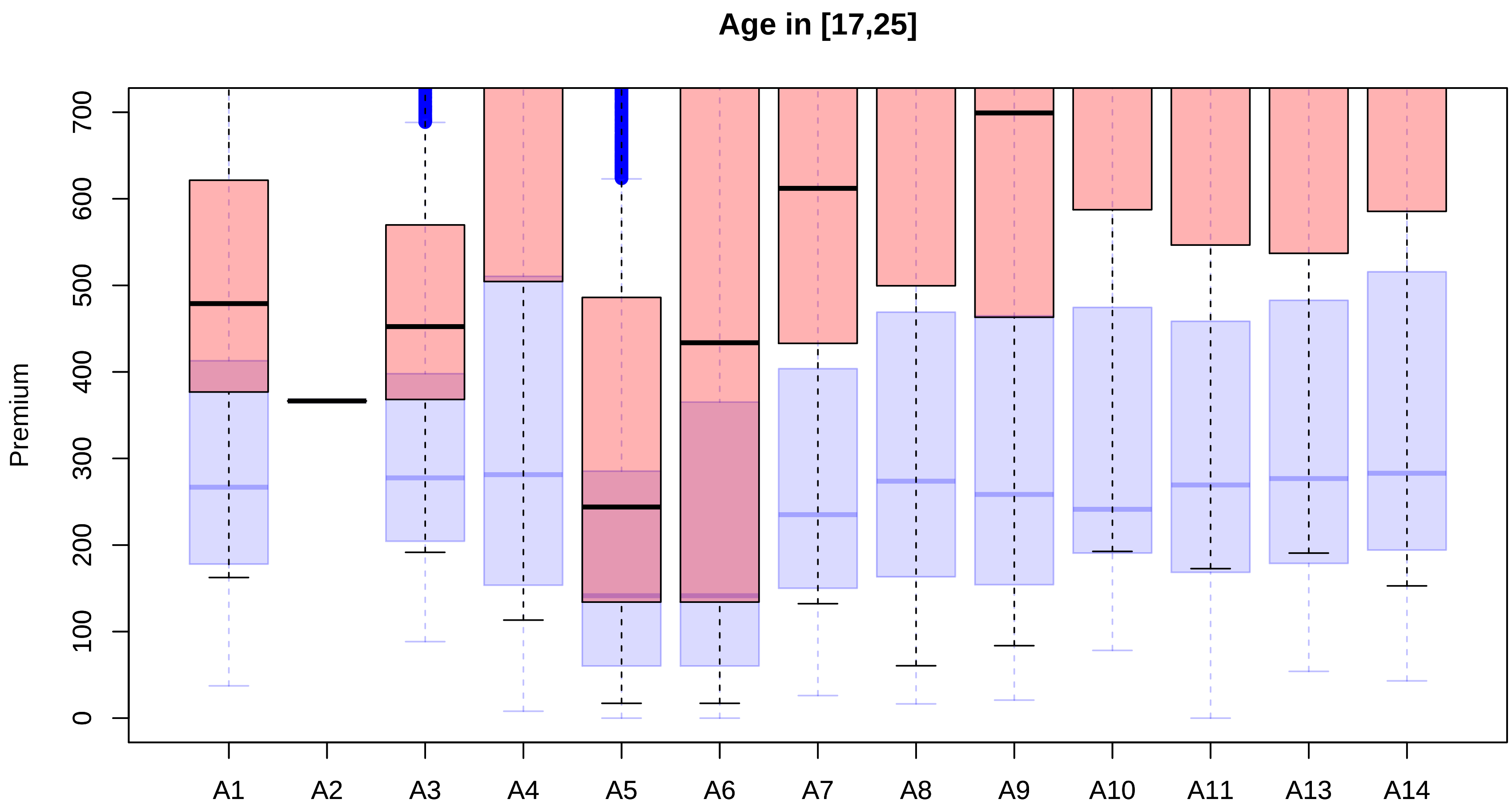

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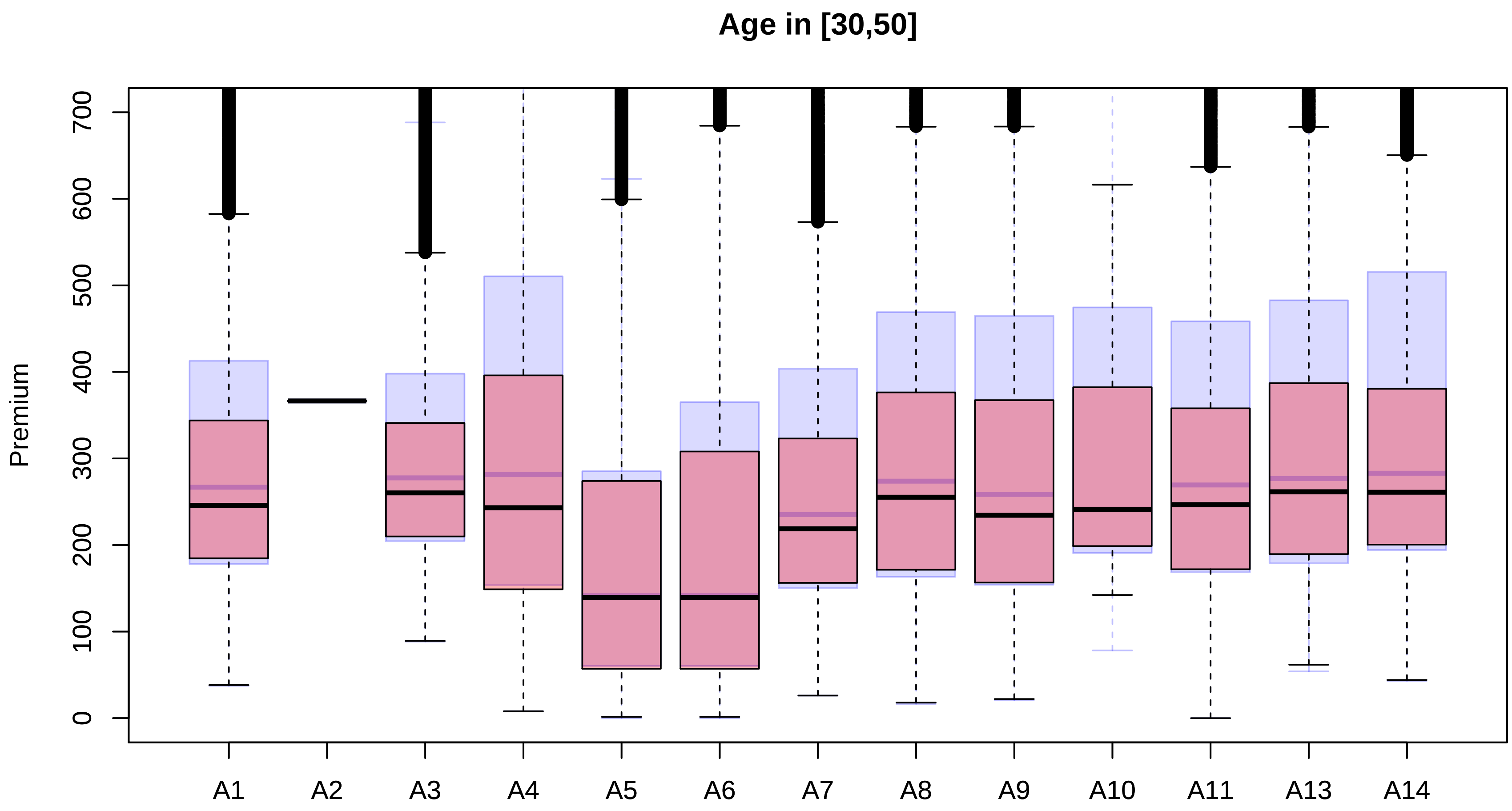

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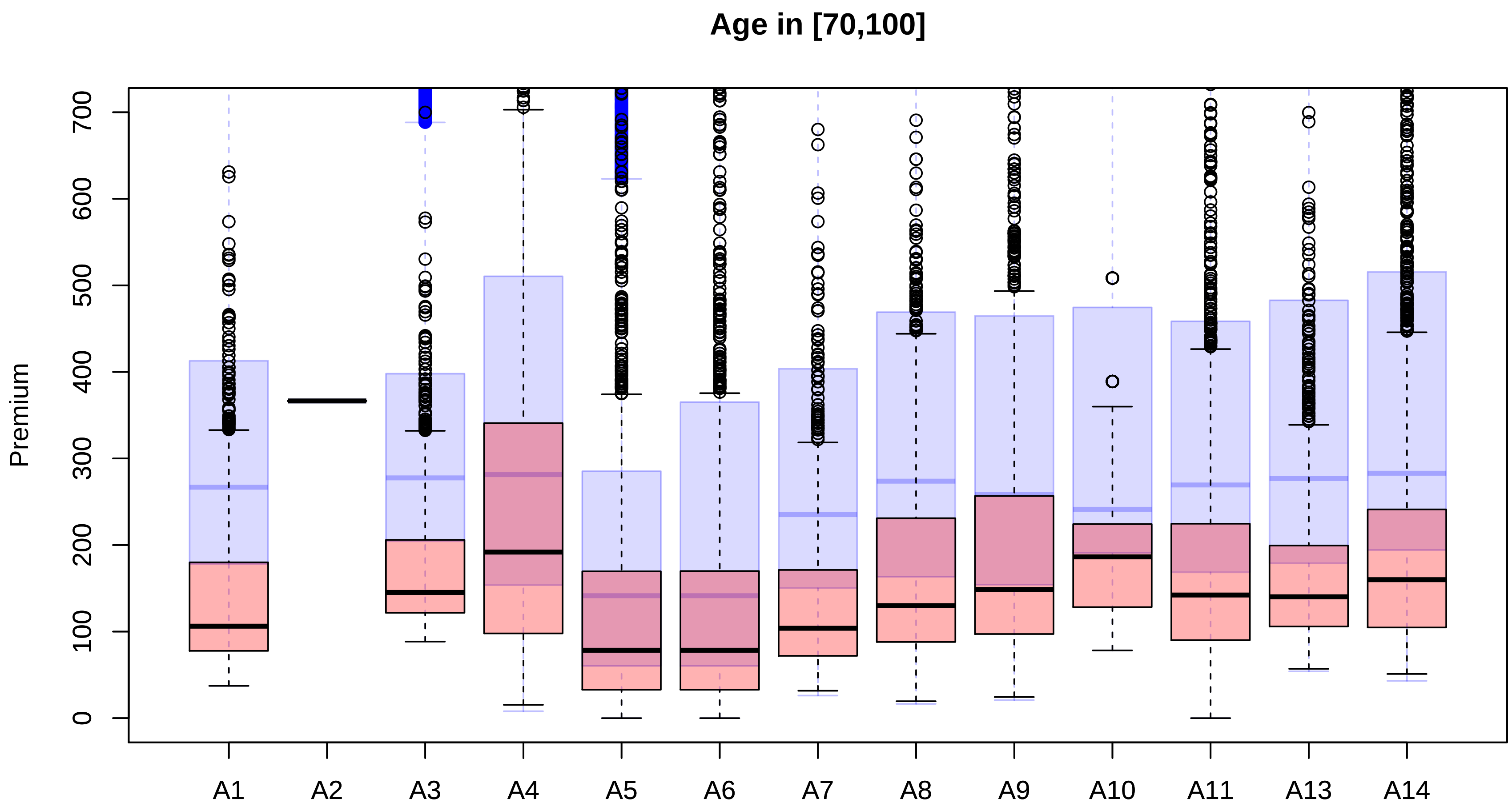

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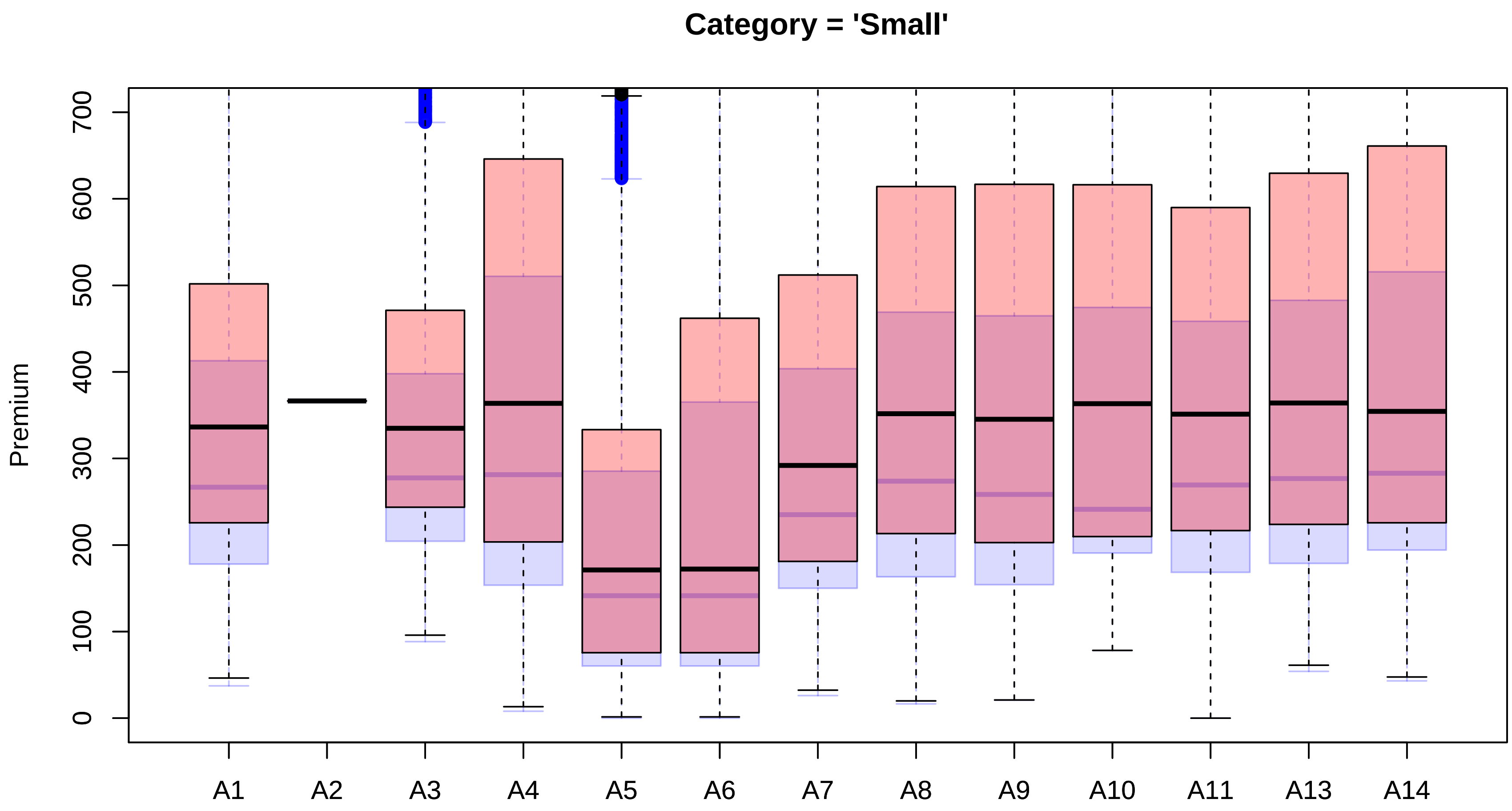

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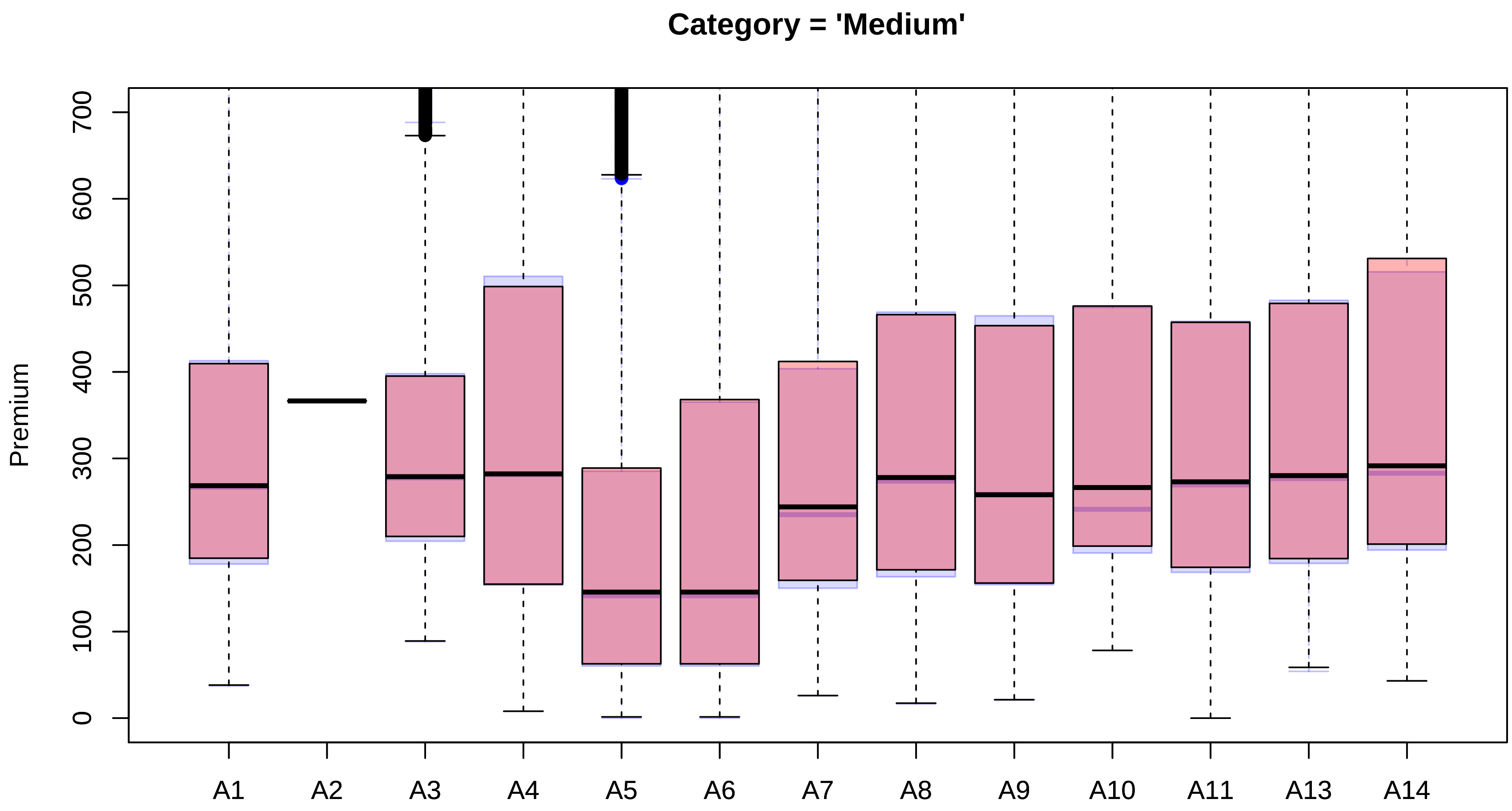

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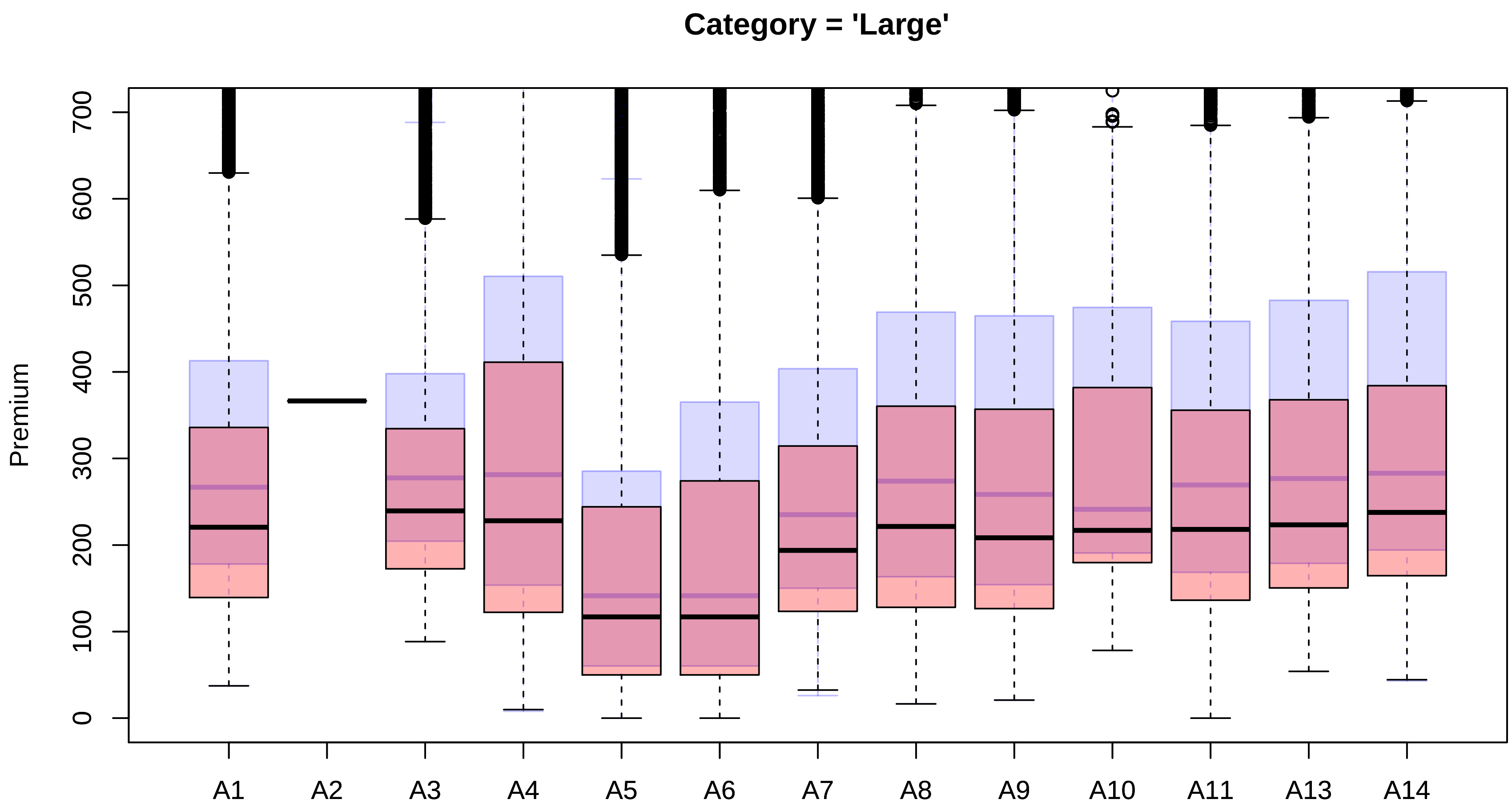

Insurance Ratemaking Competition

We need a **Decision Rule** to select premium chosen by insured i

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

Basic '**rational rule**' $\pi_i = \min\{\widehat{\pi}_1(\boldsymbol{x}_i), \cdots, \widehat{\pi}_d(\boldsymbol{x}_i)\} = \widehat{\pi}_{1:d}(\boldsymbol{x}_i)$

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

A more **realistic rule** $\pi_i \in \{\widehat{\pi}_{1:d}(\boldsymbol{x}_i), \widehat{\pi}_{2:d}(\boldsymbol{x}_i), \widehat{\pi}_{3:d}(\boldsymbol{x}_i)\}$

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
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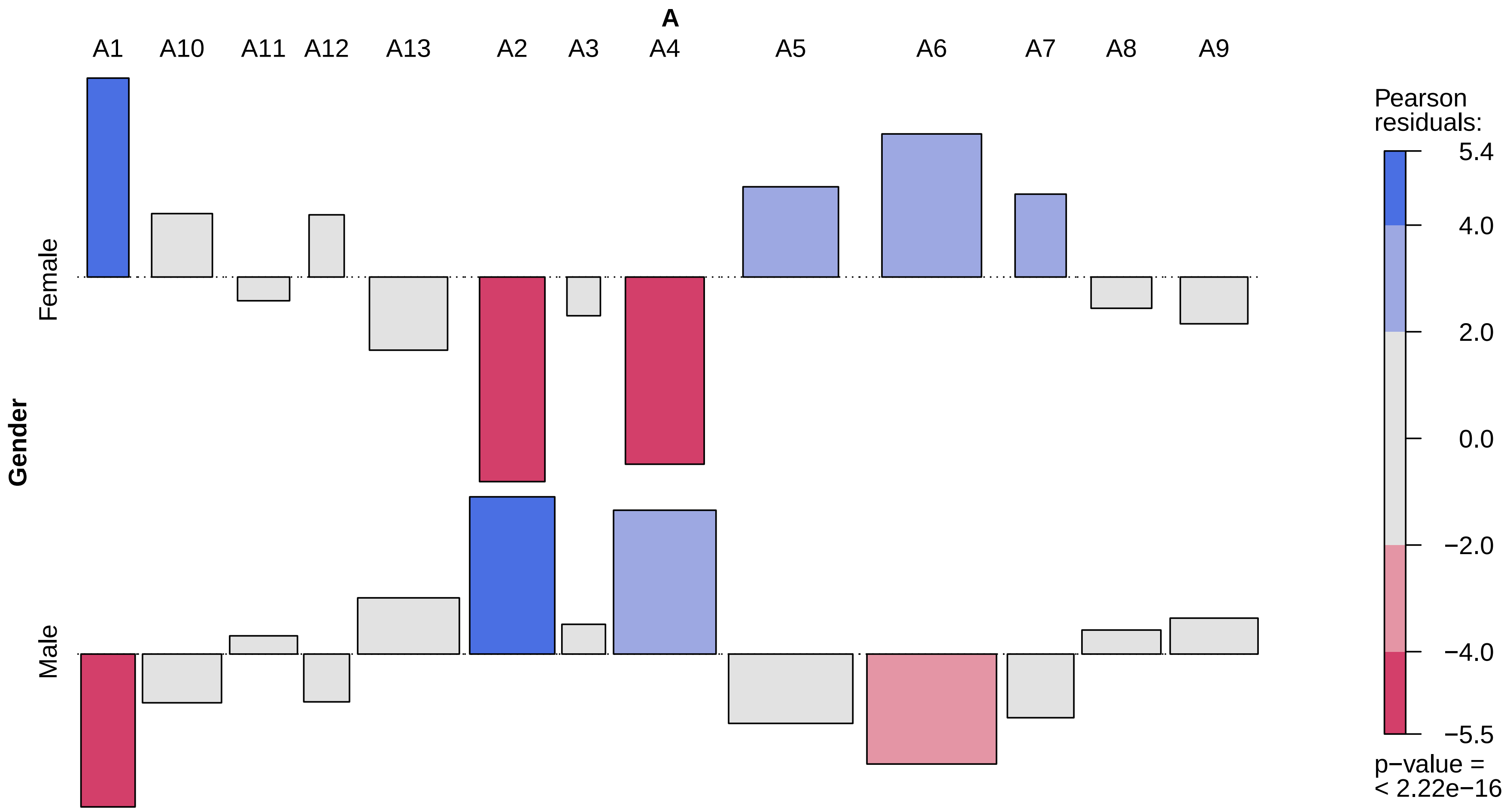
Insurance Ratemaking Competition



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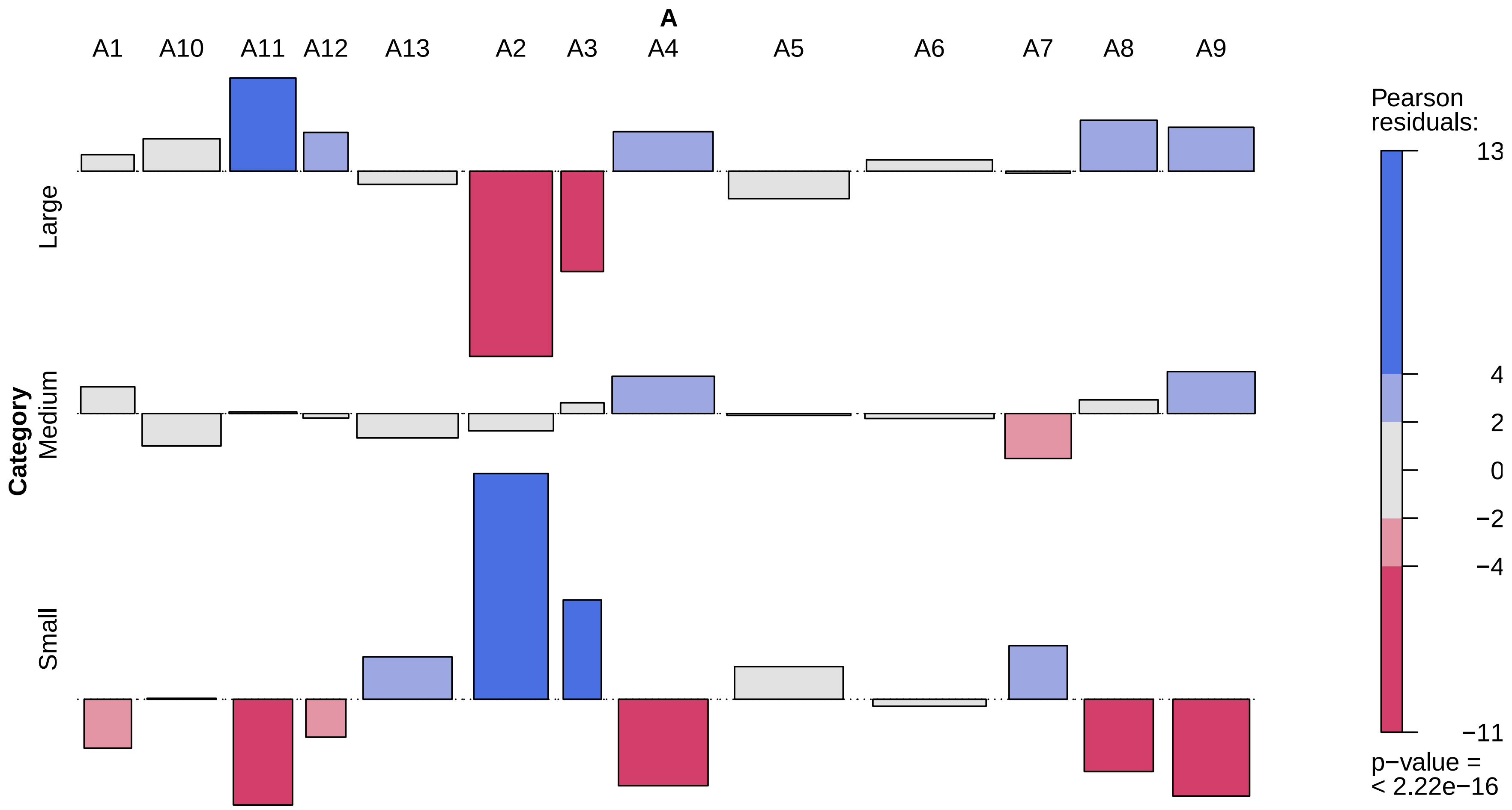

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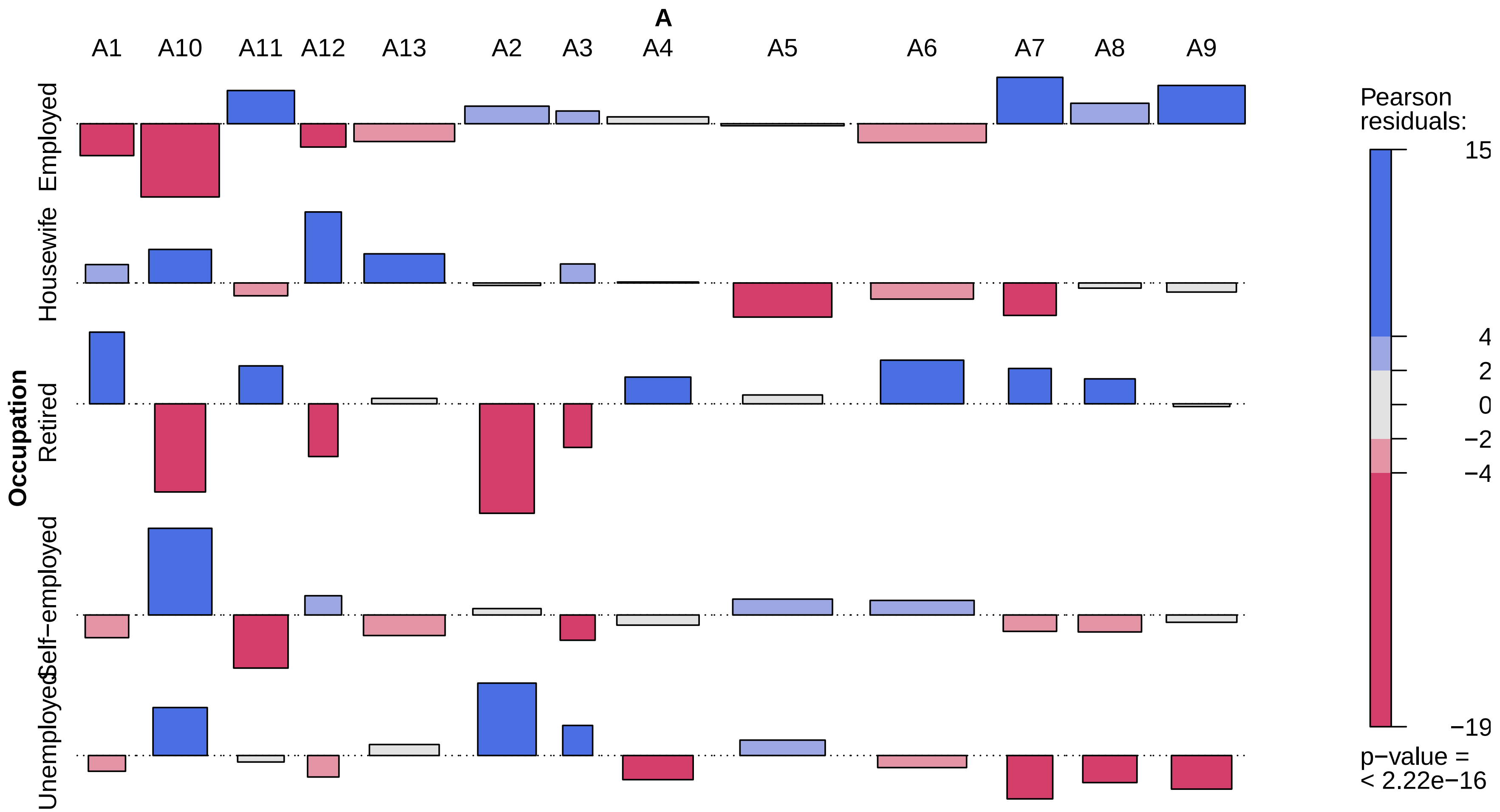

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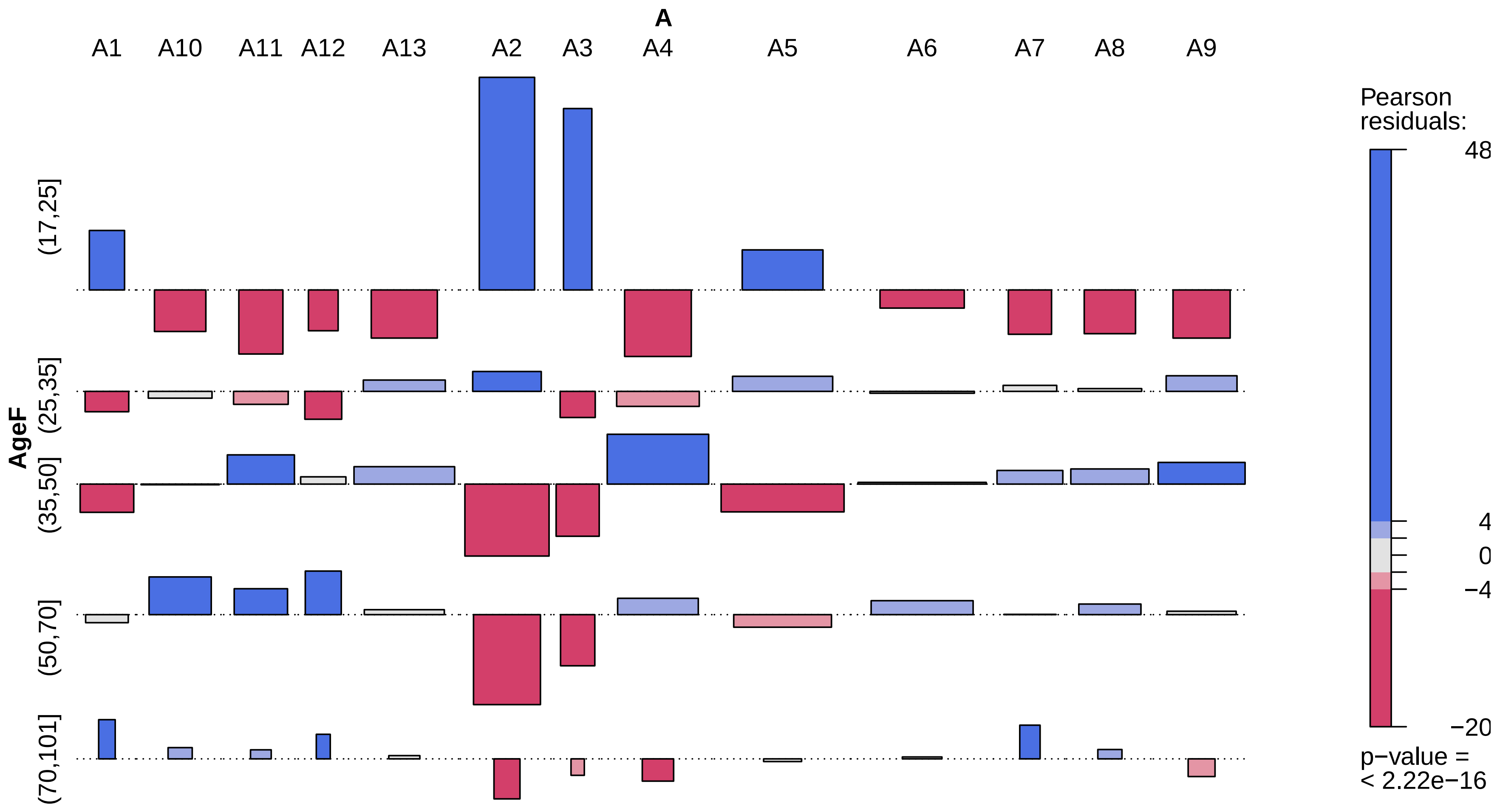


A Game with Rules... but no Goal

Two datasets : a training one, and a pricing one (without the losses in the later)

Step 1 : provide premiums to all contracts in the pricing dataset

Step 2 : allocate insured among players

Season 1 13 players

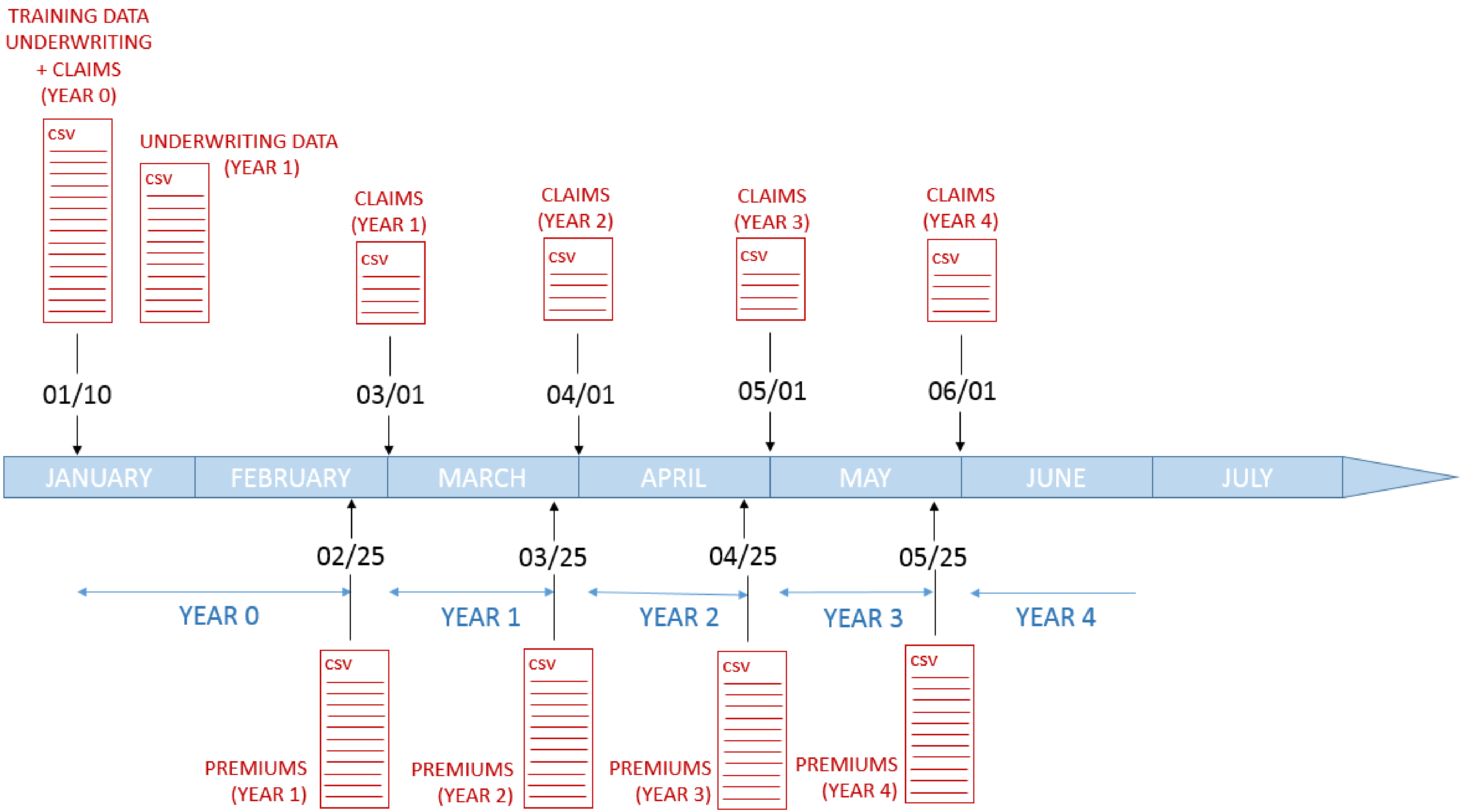
Season 2 14 players

Step 3 [season 2] : provide additional information (premiums of competitors)

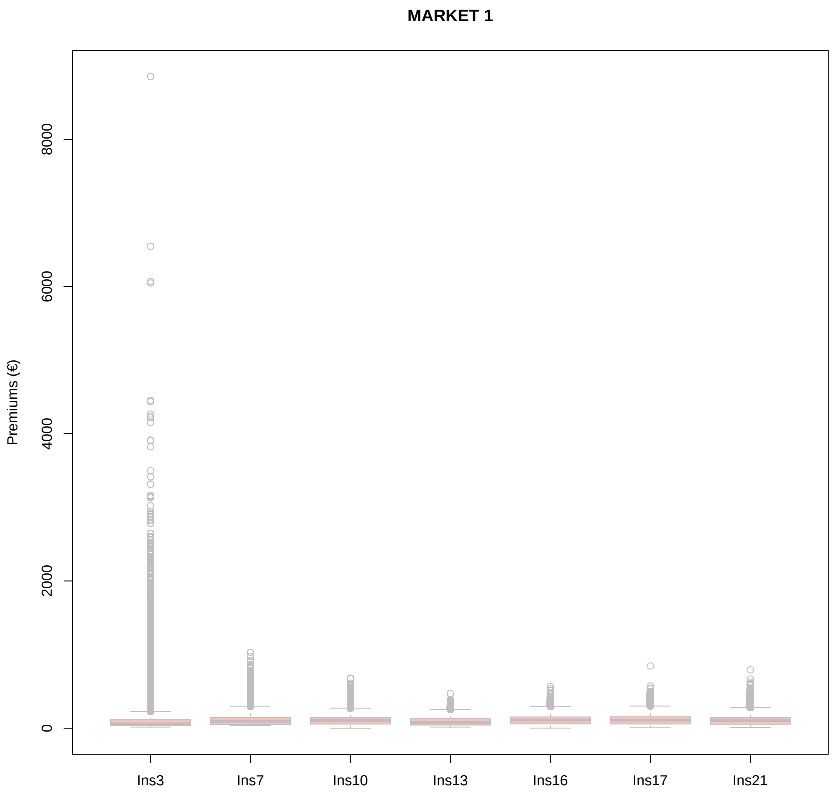
Season 3 23 players (3 markets, 8+8+7)

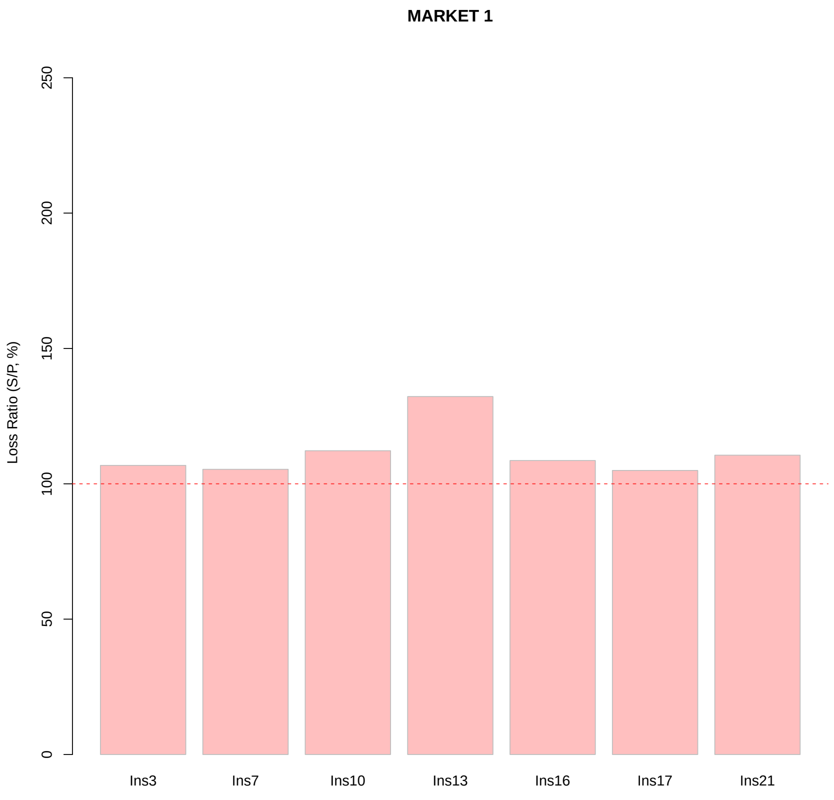
Step 3-6 [season 3] : dynamics, 4 years

Actuarial Pricing Game (season 3)



Actuarial Pricing Game (episode 1, season 3)





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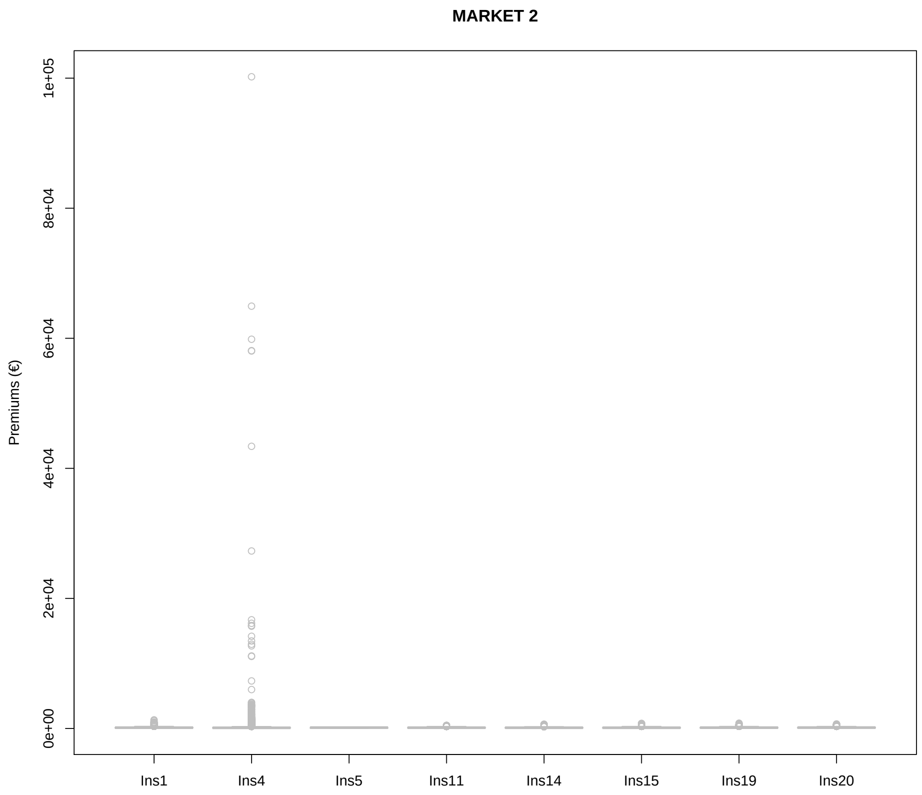

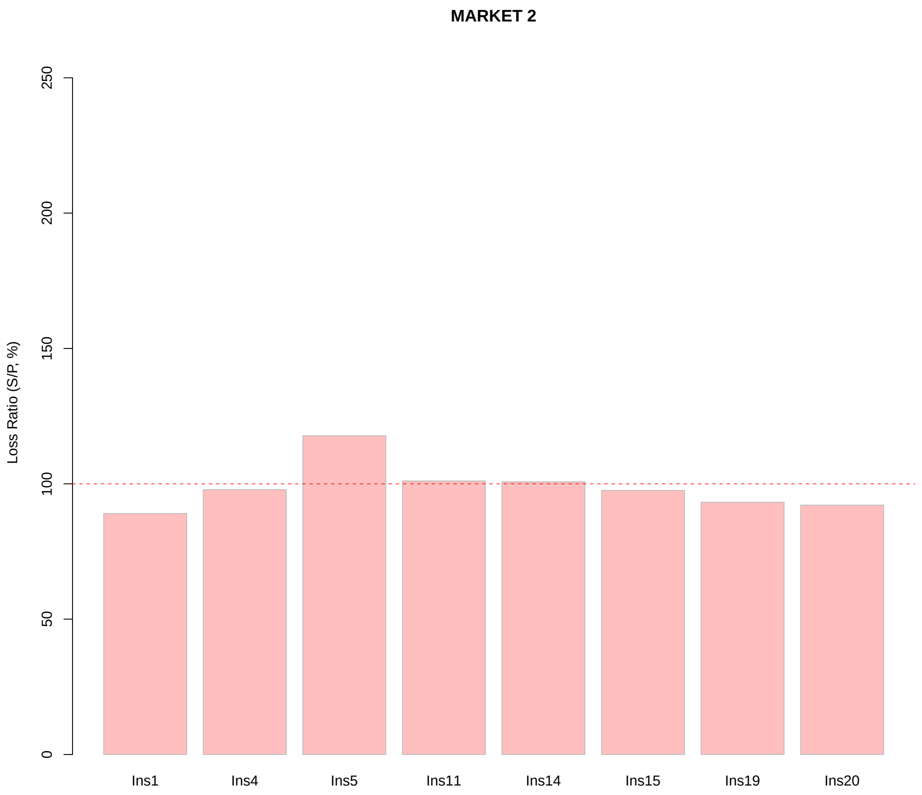

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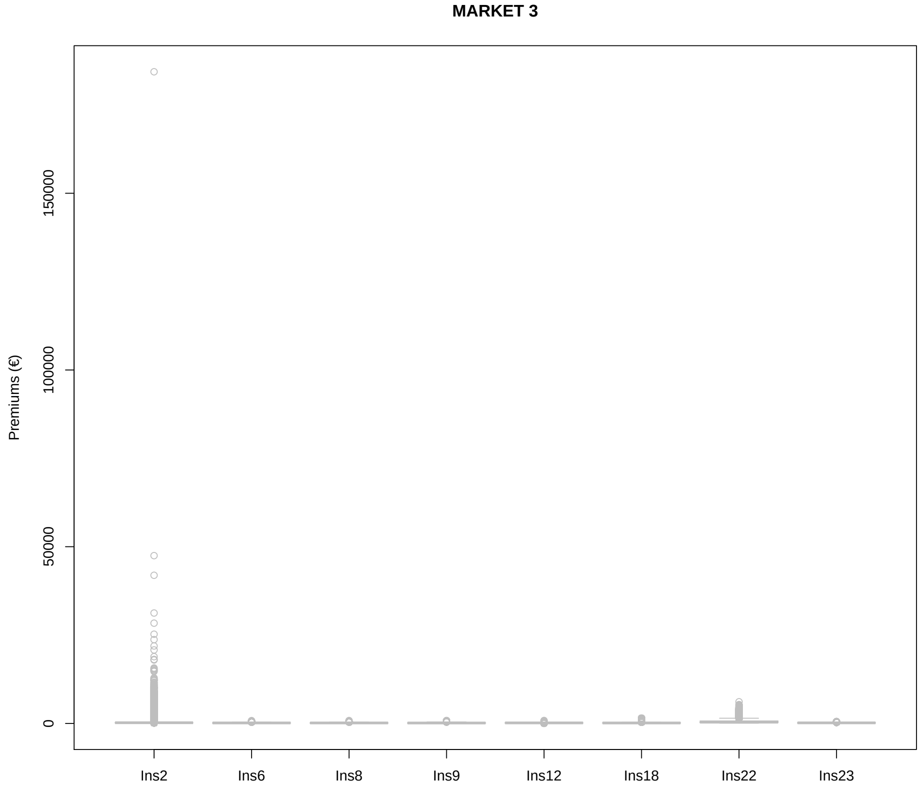

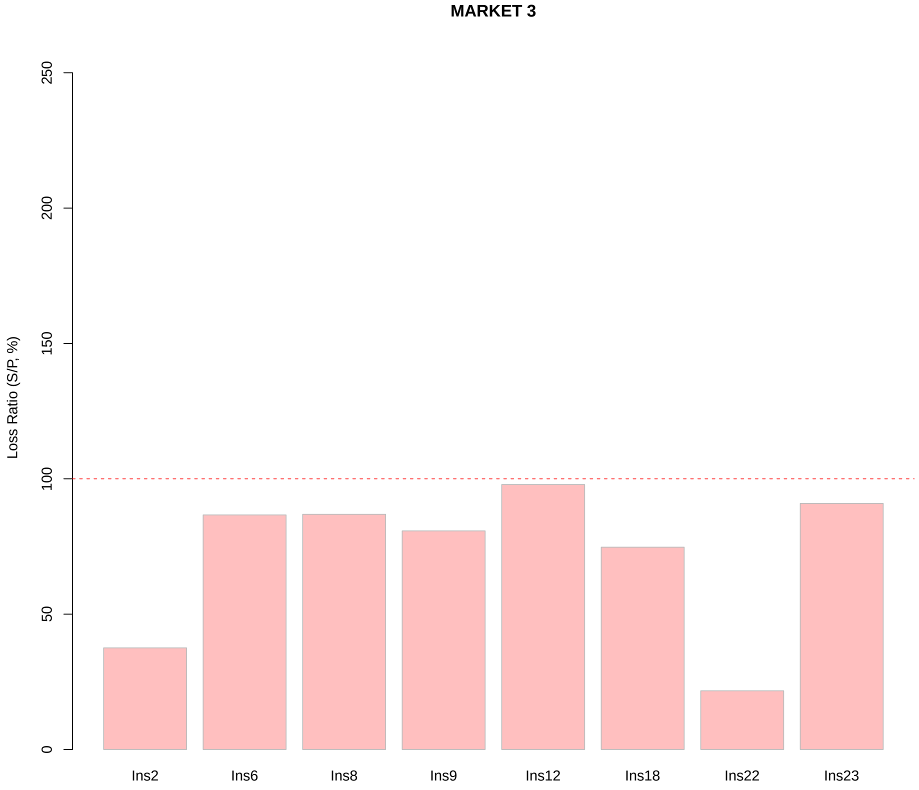

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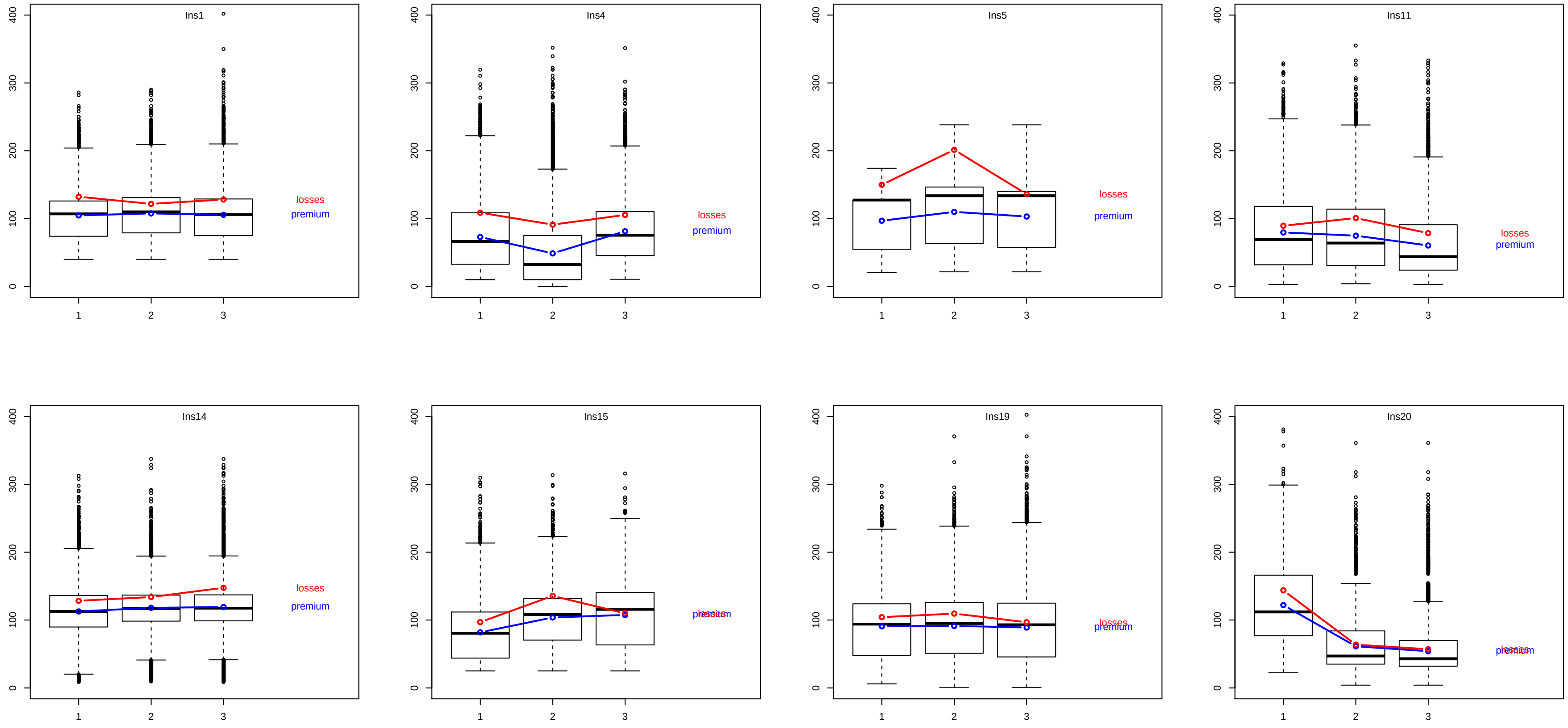

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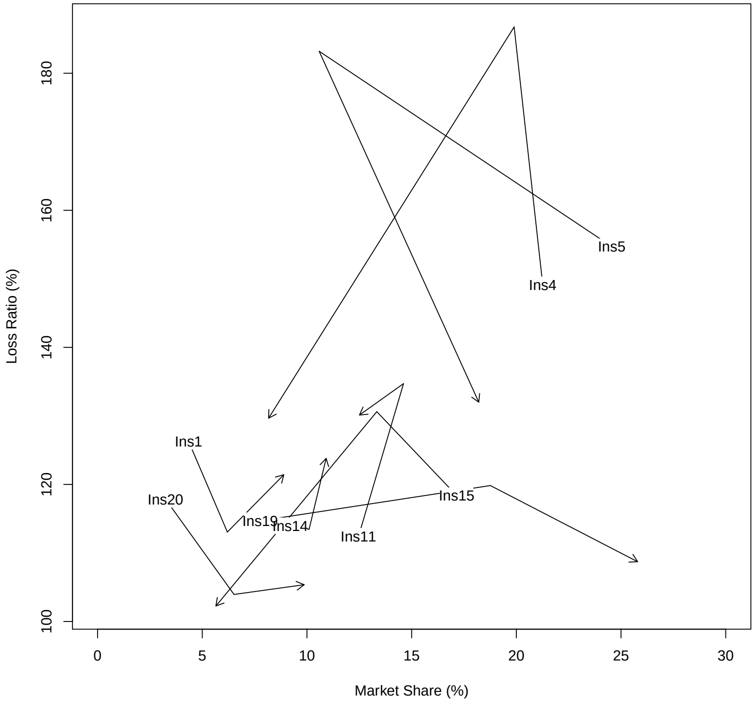

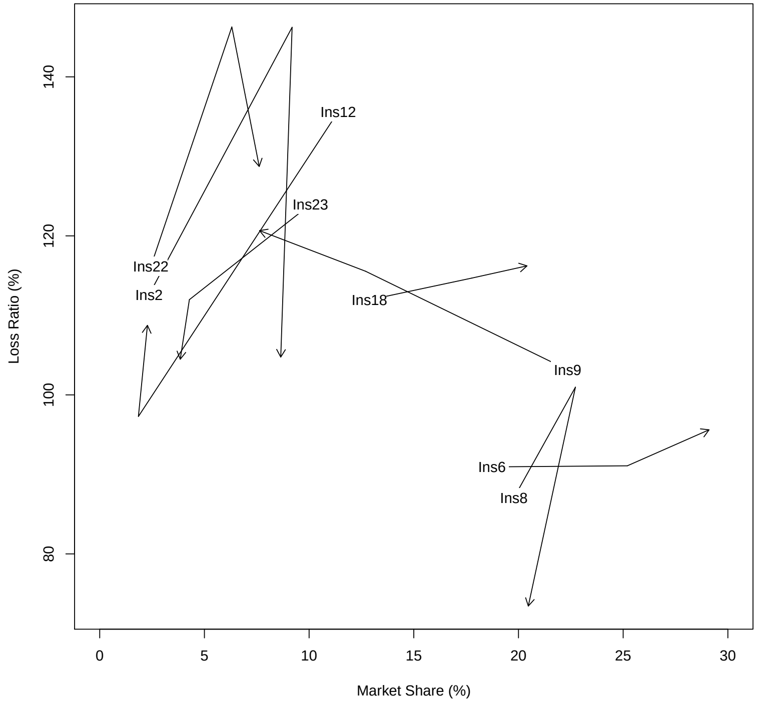

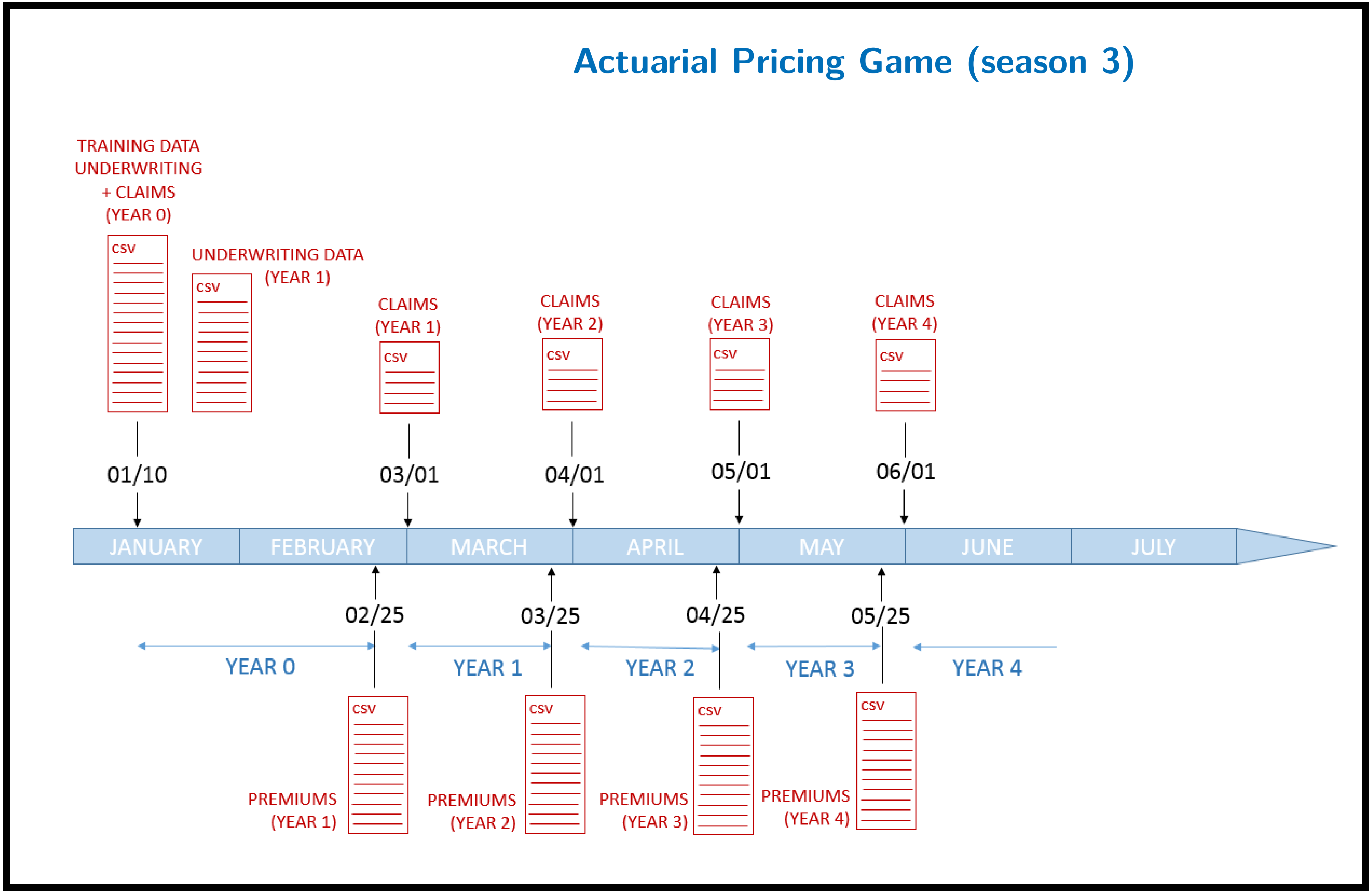
Actuarial Pricing Game (season 3)
TRAINING DATA
UNDERWRITING
+ CLAIMS
(YEAR 0)
csv
UNDERWRITING DATA
(YEAR 1)
csv
CLAIMS
(YEAR 1)
csv
CLAIMS
(YEAR 2)
csv
CLAIMS
(YEAR 3)
csv
CLAIMS
(YEAR 4)
csv
01/10
03/01
04/01
05/01
06/01
JANUARY
FEBRUARY
MARCH
APRIL
MAY
JUNE
JULY
02/25
03/25
04/25
05/25
YEAR 0
YEAR 1
YEAR 2
YEAR 3
YEAR 4
csv
PREMIUMS
(YEAR 1)
csv
PREMIUMS
(YEAR 2)
csv
PREMIUMS
(YEAR 3)
csv
PREMIUMS
(YEAR 4)